THE METHOD TO SOLVE DRIFT IN MAIN GEOMAGNETIC FIELD

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Abstract

The method to solve the velocity of drift in MGF is written in this paper. This method is based on the dynamo theory for geomagnetic field. The velocity of drift in the main geomagnetic field is also solved from the condition that the power variation of residual field equals zero in few years.

Keywords: Drift of geomagnetic field, dynamo theory for geomagnetic field

INTRODUCTION

Drift in main geomagnetic field has been studying since it is discovered by Edmund Halley in 1692. When E. Halley reconstructs map of geomagnetic field of the Earth, he observed drifts in lines with equal values of the main geomagnetic field from old maps produced in different years. He studied the drifts diligently and he obtained that mean value of the drifts was -0.5 degree for whole Earth in year. Halley's method is very easy. He measured the drifts of lines with equal values on the old maps of main geomagnetic field. Then he obtained the average of values measured. This Halley's method is being used today. Unfortunately, this Halley's method for the drift in the main geomagnetic field is not developed.[3]

Gauss C.F included the method of spherical harmonic potential to the geomagnetism from the potential theory in 1839. This method obtains the scalar potential for geomagnetic field form the Laplace's equation written for scalar potential of geomagnetic field. And values of the Gaussian coefficients in the scalar potential for geomagnetic field are detected by data measured in satellites and observatories. IGRF has also updating the

Gaussian coefficients every 5 years since 1900.

Generally, the drift in geomagnetic field is being studied by the method based on the scalar potential for geomagnetic field. I think this method is not the best advanced method for the drift in geomagnetic field.

Strong field sourced at Earth center spreads inner core, liquid core, mantle and crust and it is observed as geomagnetic field on Earth surface.

Phenomenon,which the drift in the geomagnetic field is observed about Earth's surface, is caused by fluid motions with electrical conductivity on Core Mantle Boundary (CMB).

Interaction between the strong field and fluid motions with electrical conductivity on CMB is expressed by induction equation. Form of the equation, which can study the drift in geomagnetic field about Earth surface, must have been detected.

We have obtained the form of equation about Earth surface similar to the properties of the induction equation on CMB. And Yukutake, T wrote about the form of equation in 1962,[6]. We can also solve the drift in geomagnetic field from the equation written

about the Earth surface.

The purpose of this paper is to write the

above method for researchers in easy form.

THE FORM OF INDUCTION EQUATION ABOUT EARTH'S SURFACE

Strong field is sourced by the dynamic processes in sub-shell of georeactor at the center of the Earth's inner core. [4] The strong field spreads according to inner core, liquid core, mantle and crust. And it is observed as the main geomagnetic field.

Enough intensity motions are not inner core and mantle. Therefore inner core and mantle can affect only by the diffusing in the spreading of field.

Intensity motions of conductivity liquid also exist in the liquid core. The motions can interact by the induction with the strong field.

The scale of radial component of the fluid motions in the liquid core is many times smaller than the scale of horizontal component. The reason the liquid core is in the incompressible

and unexpanded condition. And the density at bottom of liquid core is 23% more than the density of upper part of liquid core.[4] This density difference in the liquid core blocks the possibility that the fluid motions are sourced by the thermal convection in the whole liquid core. Therefore the scale of radial component of the fluid motions in the liquid core is small.

The most effectual fluid motions in the liquid core exist about the CMB. The fluid motions on CMB are horizontal motions. Its radial component is also small and it is in the turbulent form.

We can write the governing differential equation of interaction between the fluid motions and the strong field on CMB. It is written as

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{V} \times \vec{B}) + \eta \nabla^2 \vec{B} \ . \quad (1)$$

The Eq.(1) is also called the induction equation. Here η is the coefficient of magnetic diffusivity and $\vec{V}$ is the local velocity of the conductive fluid motions on the CMB.

It is possibility that the fluid motions and the field on the CMB are divided into two components such as mean motions and turbulent motions. They are written in the form

$$\left.\begin{aligned} \vec{V} &= \vec{V}_0 + \vec{v} = mean + fluctuations \\ \vec{B} &= \vec{B}_0 + \vec{b} = mean + fluctuations \end{aligned}\right\} \quad (2)$$

Velocity of the mean motions is slow varied and its L-scale is long. l-scale of the turbulent motions is small as $L \gg l$. The velocity of the turbulent motions rapidly changes and it has fluctuations. If the conditions in the Eq. (2) and $L \gg l$ the ratio in its scales are fulfilled on CMB, the dynamo theory for mean field could also used in the Eq.(1).

The westward drift is a phenomenon observed in main geomagnetic field on the Earth's surface in the large interval of space

and time. And the possibility, which the effects with small interval of space and time on CMB are being observed on the Earth's surface, is small. Thus the dynamo theory for mean field has been used for the approaching in the induction equation.

Therefore the Eq. (1) must be averaged as by Reynold's rules for averaging when the Eq.(2) is substituted into it. It can be written in the form

$$\left\langle \frac{\partial \vec{B}_0}{\partial t} + \frac{\partial \vec{b}}{\partial t} \right\rangle = \vec{\nabla} \times \left\langle (\vec{V}_0 + \vec{v}) \times (\vec{B}_0 + \vec{b}) \right\rangle + \eta \nabla^2 \left\langle \vec{B}_0 + \vec{b} \right\rangle .$$

(3)

The Eq.(3) becomes as

$$\frac{\partial \vec{B}_0}{\partial t} = \vec{\nabla} \times \left(\vec{V}_0 \times \vec{B}_0 \right) + \vec{\nabla} \times \left\langle \vec{v} \times \vec{b} \right\rangle + \eta \nabla^2 \vec{B}_0 .$$

(4)

Where $\vec{\varepsilon} = \left\langle \vec{v} \times \vec{b} \right\rangle$ is called the mean electromotive force. There is a linear relation between the mean electromotive force $\vec{\varepsilon}$ and the mean magnetic field $\vec{B}_0$. The relations are expressed in the following equation

$$\vec{\varepsilon} = \alpha \vec{B}_0 - \beta \vec{\nabla} \times \vec{B}_0 + ..., \qquad \alpha = -\frac{1}{3} \left\langle \vec{v} \cdot (\vec{\nabla} \times \vec{v}) \right\rangle \tau \quad \text{and} \quad \beta = \frac{1}{3} v^2 \tau .$$

(5)

They are called Taylor's expansion. Here α is the coefficient of the helical effects of fluid motion on the CMB, β is the coefficient for the turbulent diffusivity and τ is life-time of the helixes.

When the Eq. (5) is substituted into the Eq. (4), it could be rewritten in the following form

$$\frac{\partial \vec{B}_0}{\partial t} + (\vec{V}_0 \cdot \vec{\nabla}) \vec{B}_0 = (\vec{B}_0 \cdot \vec{\nabla}) \vec{V}_0 - \vec{B}_0 (\vec{\nabla} \cdot \vec{V}_0) + \vec{\nabla} \times \alpha \vec{B}_0 + (\eta + \beta) \nabla^2 \vec{B}_0$$

(6)

where $(\vec{V}_0 \cdot \vec{\nabla}) \vec{B}_0$ is advection term, $(\vec{B}_0 \cdot \vec{\nabla}) \vec{V}_0$ is stretching term, $-\vec{B}_0 (\vec{\nabla} \cdot \vec{V}_0)$ is compression term, $(\eta + \beta) \nabla^2 \vec{B}_0$ is diffusion and decay term. $\vec{V}_0$ is the mean velocity of the conductivity of the fluid motion on the CMB.

The core flow is usually assumed that it exists in the incompressible and unexpanded condition. Thus the stretching term $((\vec{B}_0 \cdot \vec{\nabla}) \vec{V}_0$) and the compression term $(-\vec{B}_0 (\vec{\nabla} \cdot \vec{V}_0))$ in the Eq(6) are vanished on the CMB. And the terms $\vec{\nabla} \times \alpha \vec{B}_0$ and $\beta \nabla^2 \vec{B}_0$ in the Eq.(6) show turbulent effects in the fluid motions on the CMB. The scales and the life-time τ of α and β in the Eq.(6) are small. Therefore the effects of the terms may be observed around one point or one observatory on the Earth's surface during short time. But the effects of the terms with α and β is small in the large interval of space and time. Thus the influence of the terms with α and β is very small in the same equation as Eq(6) that will be written for the surface of the Earth or in the mantle.

Above Eq(6) is only valid for the liquid core. But it can be seen from the Eq.(6) that the advection term is the main factor in the secular variation of MGF on the Earth's surface.

We can write an equation of the same kind with the Eq.(6) on the Earth's surface. The equation is

$$\frac{\partial \vec{B}}{\partial t} + (\vec{v}'_0 \cdot \vec{\nabla}) \vec{B} = \vec{R}(\vec{r}, t) ,$$

(7)

where $\vec{r}$ denotes the positional vector and $\vec{v}'_0$ is the drift velocity of the field on the Earth's surface or in the mantle. $\vec{R}(\vec{r}, t)$ is the residual field after the westward drift has been

subtracted from the observed time-variation. $\vec{R}(\vec{r},t)$ can be also described as the right hand side of the Eq.(6) but the influence of these terms is decreased by mantle. And the effects of these terms on the residual field in the Eq.(7) are observed as weaker than those in the Eq.(6) on the Earth's surface.

$$P = \int [\vec{R}(\vec{r},t)]^2 \, ds \qquad (8)$$

where ds is an interval of space and time, $\vec{R}(\vec{r},t)$ is a residual field, $[\vec{R}(\vec{r},t)]^2$ is a variance of the power density of the residual field in the small time interval and P is the power of the residual field.

The power of the residual field weakly depends on the same terms as the right hand

Therefore it is seen from the Eqs.(6-7) that the variation of MGF on the Earth's surface depends on the mean velocity of the conductivity of the fluid motion on the CMB. Now, the total power of the residual field can be rewrite in the form

side of the Eq.(6). The power variation of the strong toroidal field sourced in sub-shell in the Earth's inner core is also very small and it equals to zero in a time of 20 years.

Therefore the power variation of the residual field equals to zero in a time of 20 years as

$$\delta P = 0 . \qquad (9)$$

Once the field distribution $\vec{B}$ and its time derivative $\partial \vec{B}/\partial t$ are known, we would be able to find the velocity $\vec{v}'_0$ from the condition that the power variation of the residual field

equals to zero on the Eq.(9).

In this paper, the drift velocity $\vec{v}'_0$ of field has been solved from the condition in the Eq. (9).

To solve velocity

The power variation of the residual field in the Eq(10) could be rewritten in the form as

$$\delta P(t,r,\theta,\varphi,\dot{\theta},\omega) = \delta \int R^2(t,r,\theta,\varphi,\omega,\dot{\theta})ds = \delta \int (R_x^2 + R_y^2 + R_z^2)ds = 0 . \qquad (10)$$

The following equation as

$$R_x \cdot \frac{\partial R_x}{\partial \omega}\delta\omega + R_x \cdot \frac{\partial R_x}{\partial \dot{\theta}}\delta\dot{\theta} + R_y \cdot \frac{\partial R_y}{\partial \omega}\delta\omega + R_y \cdot \frac{\partial R_y}{\partial \dot{\theta}}\delta\dot{\theta} + R_z \cdot \frac{\partial R_z}{\partial \omega}\delta\omega + R_z \cdot \frac{\partial R_z}{\partial \dot{\theta}}\delta\dot{\theta} = 0 \qquad (11)$$

could be written from the condition of variation in the Eq. (10).
The Eq. (11) could be also rewritten in the form as

$$\left. \begin{aligned} R_x \cdot \frac{\partial R_x}{\partial \omega} + R_y \cdot \frac{\partial R_y}{\partial \omega} + R_z \cdot \frac{\partial R_z}{\partial \omega} = 0 \\ R_x \cdot \frac{\partial R_x}{\partial \dot{\theta}} + R_y \cdot \frac{\partial R_z}{\partial \dot{\theta}} + R_z \cdot \frac{\partial R_z}{\partial \dot{\theta}} = 0 \end{aligned} \right\} . \qquad (12)$$

If the derivatives as $\dfrac{\partial \vec{R}}{\partial \omega}$ and $\dfrac{\partial \vec{R}}{\partial \dot{\theta}}$ in the Eq. (12) are obtained in the analytic form, the Eq. (12) could be also rewritten in the form as

$$\left.\begin{array}{l} \dot{\theta}\cdot M_1(t,r,\theta,\varphi) + \omega\cdot P_1(t,r,\theta,\varphi) = f_1(t,r,\theta,\varphi) \\ \dot{\theta}\cdot M_2(t,r,\theta,\varphi) + \omega\cdot P_2(t,r,\theta,\varphi) = f_2(t,r,\theta,\varphi) \end{array}\right\} \qquad (13)$$

Where $M_1(t,r,\theta,\varphi)$, $P_1(t,r,\theta,\varphi)$, $f_1(t,r,\theta,\varphi)$, $M_2(t,r,\theta,\varphi)$, $P_2(t,r,\theta,\varphi)$ and $f_2(t,r,\theta,\varphi)$ are analytic functions depended on t, r, θ and φ.

The distribution functions of angular velocities of field drift $\dot{\theta}(t,r,\theta,\varphi)$ and $\omega(t,r,\theta,\varphi)$ could be solved in the form as

$$\left.\begin{array}{l} \dot{\theta}(t,r,\theta,\varphi) = \dfrac{f_1(t,r,\theta,\varphi)\cdot P_2(t,r,\theta,\varphi) - f_2(t,r,\theta,\varphi)\cdot P_1(t,r,\theta,\varphi)}{M_1(t,r,\theta,\varphi)\cdot P_2(t,r,\theta,\varphi) - M_2(t,r,\theta,\varphi)\cdot P_1(t,r,\theta,\varphi)} \\[4mm] \omega(t,r,\theta,\varphi) = \dfrac{M_1(t,r,\theta,\varphi)\cdot f_2(t,r,\theta,\varphi) - M_2(t,r,\theta,\varphi)\cdot f_1(t,r,\theta,\varphi)}{M_1(t,r,\theta,\varphi)\cdot P_2(t,r,\theta,\varphi) - M_2(t,r,\theta,\varphi)\cdot P_1(t,r,\theta,\varphi)} \end{array}\right\} \qquad (14)$$

from the system of equations in the Eq(13).

The average values of angular velocities of field drift can be solved how they are averaged by the space and time as

$$\dot{\theta} \approx \frac{\int \dot{\theta}(t,r,\theta,\varphi)ds}{\int ds} \quad \text{and} \quad \omega \approx \frac{\int \omega(t,r,\theta,\varphi)ds}{\int ds}. \qquad (15)$$

If integrals in the Eq. (15) are difficult to integrate, the easy polynomial forms as the Eq. (16) could be also built by the values solved via the Eq. (14).

AN EXAMPLE

On the Europe, we have obtained distribution functions of space and time for main geomagnetic field such as

$$B_x = C_{00} + C_{01}\varphi + C_{02}\varphi^2 + C_{03}\varphi^3 + C_{10}\theta + C_{11}\theta\varphi + C_{12}\theta\varphi^2 + C_{20}\theta^2 + C_{21}\varphi\theta^2 + C_{30}\theta^3 \qquad (16)$$

where φ is longitude, θ is latitude, and C_{00}, … C_{30} are polynomial coefficients. The dependences on time in the polynomial coefficients for components B_X, B_Y, and B_Z, are shown on the table 1.

The functions as the Eq. (16) are also built by the values of main geomagnetic field abstracted from data of observatories located on the Europe. Possibility levels of the functions as the Eq. (16) are about 97-99 percent by the reduced chi-square test.

The functions as the Eq. (16) are also obtained in the NED system of coordinate but the velocity of field drift is solved in the coordinate system of ECEF. Therefore the distribution functions in the Eq. (16) for NED must be transformed to ECEF but the quadrate of a vector is not varied by the inversion operation or the rotation of coordinate system on the Earth's surface. It is expressed as

$$[\vec{R}_e(\vec{r},t)]^2 = [\hat{I}\frac{\partial \vec{B}}{\partial t} + \hat{I}(\vec{v}'_0\cdot\vec{\nabla}_e)\vec{B}]^2 = \hat{I}^2[\frac{\partial \vec{B}}{\partial t} + (\vec{v}'_0\cdot\vec{\nabla}_e)\vec{B}]^2 = [\frac{\partial \vec{B}}{\partial t} + (\vec{v}'_0\cdot\vec{\nabla}_e)\vec{B}]^2, \qquad (17)$$

where $\hat{I}$-is the inversion operator from NED to ECEF, $\vec{\nabla}_e$ -is Nable's operator in ECEF. Thus

we can write components of the residual field as

$$R_{ex}(\vec{r},t) = \frac{\partial B_x}{\partial t} + [v'_{0x}\frac{\partial B_x}{\partial x} + v'_{0y}\frac{\partial B_x}{\partial y} + v'_{0z}\frac{\partial B_x}{\partial z}].$$

(18)

Table 1.

	Europe		
	B_x	B_y	B_z
C_{00}	-0.79t³+28.22t²-253.34t+ 49320.94	-0.601t³+24.32t²- 269.764t+1814.07	-2.81t³+86.55t²-730.61t- 42252.38
C_{01}	-290.164t+6472.407	-1.122t³⁺32.23t²-162.8t+4613.3	-4.596t³+140.107t²- 833.83t- 1043.21
C_{02}	212.34t+5017.157	-6.53t²+315.95t- 16129.51	33.78t²-1224.01t+ 15682.71
C_{03}	-0.048t³-1.42t²+34.98t-681.798	-2.48t²+96.67t+655.054	-5.79t²+148.61t-3773.10
C_{10}	2.611t³-91.474t²+922.32t- 38396.769	2.22t³-85.49t²+ 1037.27t-7733.42	9.59t³-308.56t²+2816.7t+ 192757.05
C_{11}	453.386t-16664.086	-446.87t+11705.44	9.24t³-298.33t²+2267.3t+8822.88
C_{12}	-191.376t-6551.478	-228.86t+12840.865	-1.88t³+16.5t²+ 777.27t- 7568.86
C_{20}	-2.751t³+95.57t²-1006.358t+5209.597	-2.72t³+ 98.78t²-1096.7t+5678.08	-10.76t³+357.11t²-3361.87t- 142531.1
C_{21}	-200.73t+10926.704	304.28t-8005.95	-4.55t³+156.06t²- 1390.52t- 7658.87
C_{30}	0.945t³-32.644t²+ 349.745t+ 73.998	1.059t³-36.48t²+ 366.02t- 1303.48	3.91t³-132.97t²+1283.62t+ 38877.98

And the derivatives in the Eq. (18) as $\frac{\partial B_x}{\partial x} = \frac{\partial B_x}{\partial \varphi}\frac{\partial \varphi}{\partial x} + \frac{\partial B_x}{\partial \theta}\frac{\partial \theta}{\partial x}$ could be solved by the Eq.

(16) and the easy trigonometric ratios as

$$\frac{\partial \varphi}{\partial x} = \frac{\partial}{\partial x}\arccos(\frac{x}{\sqrt{x^2+y^2}}) = -\frac{\sin\varphi}{r\cos\theta}, \frac{\partial \theta}{\partial x} = -\frac{\cos\varphi\sin\theta}{r}, \frac{\partial \varphi}{\partial y} = \frac{\cos\varphi}{r\cos\theta}, \frac{\partial \varphi}{\partial z} = 0,$$

$$\frac{\partial \theta}{\partial x} = -\frac{\cos\varphi\sin\theta}{r}, \frac{\partial \theta}{\partial y} = -\frac{\sin\varphi\sin\theta}{r}, \text{and } \frac{\partial \theta}{\partial z} = \frac{\cos\theta}{r}.$$

(19)

The components of velocity are expanded as

$$v'_{0x} = -r\sin\theta\cos\varphi\cdot\dot{\theta} - r\cos\theta\sin\varphi\cdot\omega.$$

(20)

The Eq.(18) and Eq.(20) must be substituted into the Eq. (12). Then the form as the Eq. (13) is obtained from the Eq. (12). And the distribution functions of angular velocities of

field drift in the Eq.(14) must be solved from the Eq.(13). We can also solve the average velocities of field drift on the Europe as

$$\dot{\theta}_{eu} \approx \frac{\int_{1991}^{2006}\int_0^{R_0}[\int_{\pi/6}^{4\pi/9}\int_{-\pi/18}^{2\pi/9}\dot{\theta}_{eu}(t,r,\theta,\varphi)\cdot r^2\cos\theta d\varphi d\theta drdt}{\int_{1991}^{2006}\int_0^{R_0}[\int_{\pi/6}^{4\pi/9}\int_{-\pi/18}^{2\pi/9}r^2\cos\theta d\varphi d\theta drdt} \approx -0.02\deg ree/year$$

(21)

and

$$\omega_{eu} \approx \frac{\int\limits_{1991}^{2006} \int\limits_{0}^{R_0} [\int\limits_{\pi/6}^{4\pi/9} \int\limits_{10\pi/9}^{2\pi/9} \omega_{eu}(t,r,\theta,\varphi) \cdot r^2 \cos\theta d\varphi d\theta dr dt}{\int\limits_{1991}^{2006} \int\limits_{0}^{R_0} [\int\limits_{\pi/6}^{4\pi/9} \int\limits_{-\pi/18}^{2\pi/9} r^2 \cos\theta d\varphi d\theta dr dt} \approx -0.303 \deg ree / year \cdot \tag{22}$$
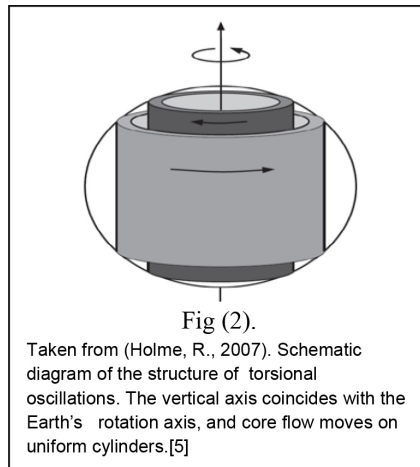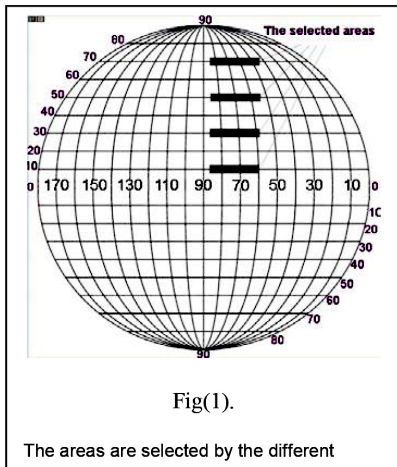
CONCLUSIONS

The advection term $(\vec{V}_0 \cdot \vec{\nabla})\vec{B}_0$ in the Eq. (6) is the main factor in the time variation or the secular variation of MGF. In other words, the Eq. (6) for the CMB very weakly depends on the terms in the right hand side. And the variation does not exist in the power of the strong field in a few years. These are the theoretical basis of this method to define the drift in geomagnetic field.

We could predict that the power of MGF for whole Earth is steady in few years and the variation in MGF on points of Earth's surface is sourced by the movement of field.

If any latitudinal dependence exists in the drift of geomagnetic field, it must be detected clearly by this method on the selected areas in the fig (1). And some models for MGF as shown on the fig (2) could be tested by this latitudinal dependence on the drift of geomagnetic field.

Fig(1).

The areas are selected by the different

Fig (2).

Taken from (Holme, R., 2007). Schematic diagram of the structure of torsional oscillations. The vertical axis coincides with the Earth's rotation axis, and core flow moves on uniform cylinders.[5]

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