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ARTICLE

Beyond likelihood-Based Inference: A Detailed Bayesian Paradigm for Heavy-Tailed Reliability data

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Abstract: This study explores the Bayesian analysis of Modified Logistic Lomax distribution. Markov Chain Monte Carlo (MCMC) is used for the parameter estimation using a real dataset. Convergence Diagnostics, Predictive performances and Inferential Survival Characteristics are analysed. The P-P and Q-Q plots, as well as the Posterior Survival plot and Hazard rate plots demonstrate excellent fitting of the distribution under Bayesian parameter estimations. The model demonstrated excellent convergence (R-hat < 1.01) and superior predictive accuracy (WAIC = 917.4). The 95% credible intervals for Mean Survival Time (128.52-137.20) practically supports the utility of this framework. This study contributes to the Bayesian literature on probability and inferential statistics as well as to the future researcher for analysing real data sets using Bayesian approaches. All the computational analysis of the study is performed using R-language programming.

Keyword: *Bayesian approach, Convergence diagnostics, Markov chain Monte Carlo (MCMC), Predictive performance, Survival analysis;*

INTRODUCTION

The cornerstone of engineering, actuarial, biostatistics, public health and medicine calls for exact statistical modeling for reliability, as well as survival data analysis. Such data has a very common and important, but critical characteristic, which is the presence of heavy tailed behaviour of modern data sets found in various fields. These characteristics and requirement result in probability analysis of extreme and long-term failure events that are significantly and precisely higher than in standard distributions. For decision making, risk assessment, warranty analysis, quality assurance, and predictive maintenance require effective capturing of these behaviours.

Traditional methods of parameter estimations, like maximum likelihood Estimation (MLE) and least square methods (LSE) offer limited valuable insights while analysing these data.These models often fail to quantify estimation uncertainty as well as to provide intuitive probabilistic interpretations for forecasting and estimations, which motivates a paradigm shift towards the most effective and significant approaches like Bayesian Inferences. Bayesian approach, which uses prior information from historical data, is a powerful alternative to improving estimates of the parameters, when data is scarce and uncertain. This enables researchers and analysts to give a complete and effective posterior distribution for parameters offering accurate, as well as precise measures of uncertainty providing credible intervals for estimated parameters. This study moves beyond the likelihood-based inferences to demonstrate a detailed Bayesian paradigm generally designed for heavy-tailed dataset modeling under probability distribution applying on real data analysis.

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There is very detailed literature review regarding the Bayesian estimation of parameter and inferential statistics. In modern statistics, study of Bayesian approach can be studied in categorising the following: (i) the development of flexible and precise model under study; (ii) classical frequency based inferential method of data analysis, and (iii) the application and support of Bayesian Analysis. Various standard probability models, like Weibull distribution [1], Half Cauchy modified [2], modified exponential [3] Generalised Rayleigh [4], and log-normal [5] significantly fit the skewed and heavy tailed datasets significantly. Apart from these models, there are many probability models like gamma [6] and generalised gamma [7] that fit these data better. Due to non-monotonic hazard rate function, Logistic-Lomax distribution [8] and its extension are focused mostly in literature. Generalisation, exponentiations and modification are various techniques of getting more flexible models from the classical models. Our previous work introduced a novel probability model called Modified Logistic-Lomax [9] model, capable of capturing heavy-tailed behaviour and outperforming several frequentist-based probability models.

The estimation of parameters for these complex models is mainly based on frequentist approaches like maximum likelihood estimation and bootstrapping, which become less reliable for small sample size and result in misinterpreting confidence interval. [10] explains this paradigm providing a foundation for model's properties and frequentist estimation.

The applications of Bayesian methods for survival and reliability analysis are driven by advances in computational Markov chain Monte Carlo (MCMC) techniques. Bayesian approaches have been applied to standard model like Weibull [11] Modified exponential, and Modified Inverse Lomax [12] and other complex models. These studies significantly provide better ability to handle prior information, they have more interpretable confidence intervals and are applicable for small sample size. Study of Bayesian analysis for modified heavy-tailed distributions, along with comprehensive convergence diagnostics, posterior and predictive checks remain an area for further exploration.

While Bayesian methods have been applied to standard classical models, as well as custom models, a comprehensive Bayesian procedure - including detailed convergence diagnostics, posterior predictive checks, and survival characteristic inference have not been applied on newly proposed MLL model. This study addresses this gap in modern literature.

This study seeks to address these gaps by providing a complete Bayesian framework for studying and analysing heavy-tailed reliability data using Modified Logistic-Lomax Distribution.

MATERIAL AND METHODS

In this section of the study, we have performed the Bayesian analysis of a custom model Modified Logistic-Lomax [9] distribution.

Modified Logistic-Lomax Distribution

The probability distribution function, probability density function and log likelihood function of the MLL model are demonstrated in equation 1 to 3.

$$F(x,\alpha,\beta,\lambda)=1-\frac{1}{1+\left[\left(1+\beta e^{\lambda x}\right)-1\right]^{\alpha}};x>0,(\alpha,\beta,\lambda)>0 \tag{1}$$

$$f(x,\alpha,\beta,\lambda)=2\alpha\lambda\beta e^{\lambda x}\left(1+\beta e^{\lambda x}\right)\left(\left(1+\beta e^{\lambda x}\right)^{2}-1\right)^{\alpha-1}\left(1+\left(\left(1+\beta e^{\lambda x}\right)^{2}-1\right)^{\alpha}\right)^{-2},x>0 \tag{2}$$

$$\ell(\alpha,\beta,\lambda\mid x)=n\log 2+n\log\alpha+n\log\beta+n\log\lambda+\lambda\sum_{i=1}^{n}x_i+\sum_{i=1}^{n}\ln\left(1+\beta e^{\lambda x_i}\right)+ (\alpha-1)\sum_{i=1}^{n}\ln\left[\left(1+\beta e^{\lambda x_i}\right)^2-1\right]-2\sum_{i=1}^{n}\ln\left[1+\left\{\left(1+\beta e^{\lambda x_i}\right)^2-1\right\}^{\alpha}\right] \tag{3}$$

The corresponding hazard rate function is given in Equation 4.

$$h(x)=2\alpha\lambda\beta e^{\lambda x}\left(1+\beta e^{\lambda x}\right)\left(\left(1+\beta e^{\lambda x}\right)^2-1\right)^{\alpha-1}\left(1+\left(\left(1+\beta e^{\lambda x}\right)^2-1\right)^{\alpha}\right)^{-1},x>0 \tag{4}$$

The variations in probability density plots (Fig 1, left) and hazard rate plots (Fig 1, right) for different sets of parameters supports the flexibility of the model.

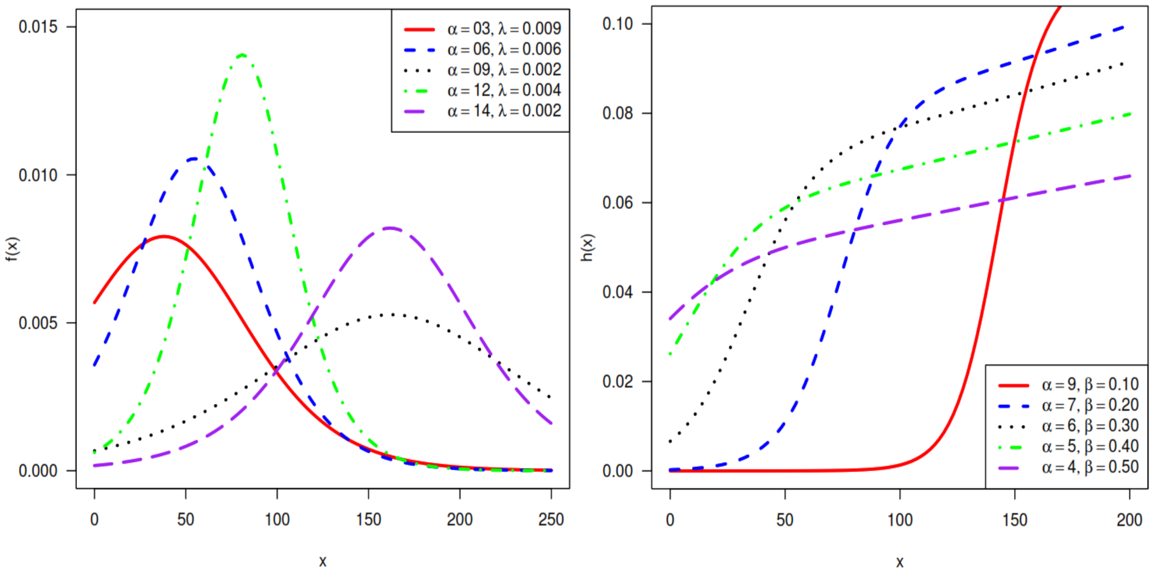

Figure 1: Density plots for β = 0.3 (left) and hazard rate plots for λ = 0.05 (right).

To check the applicability of the model, we have considered a dataset. Originally [13] analysed the data given below which represents the fatigue life of 6061-T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second (cps) which consists of 101 observations with maximum stress per cycle 31,000 psi. Parameters of the model are estimated using optim() function of R software [14].

Bayesian Analysis

Under Bayesian analysis, we have defined the likelihood function, which is the product of individual densities in equation 5

$$L(x\mid\alpha,\beta,\lambda)=\prod_{i=1}^{n}f\left(x_i\mid\alpha,\beta,\lambda\right) \tag{5}$$

Independent Gamma priors are chosen for parameters and are given as;
α ~ Gamma (0.1, 0.1)
β ~ Gamma (0.1, 0.1)
λ ~ Gamma (0.1, 0.1)
These weakly informative Gamma priors were used to allow the data to dominate the posterior while providing numerical stability. The joint posterior distributions of the parameters for given data are proportional to the products of likelihood and priors, given in equation (6).

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$$\pi(\alpha,\beta,\lambda \mid x) \propto L(x \mid \alpha,\beta,\lambda).\pi(\alpha).\,\pi(\beta).\,\pi(\lambda) \tag{6}$$

Substituting the defined models in equation (6) give complete posterior distribution defined in equation (7).

$$\pi(\alpha,\beta,\lambda \mid x) \propto \prod_{i=1}^{n} 2\alpha\lambda\beta e^{\lambda x_i}\left(1+\beta e^{\lambda x_i}\right)\left(\left(1+\beta e^{\lambda x_i}\right)^2-1\right)^{\alpha-1}$$

$$\left(1+\left(\left(1+\beta e^{\lambda x_i}\right)^2-1\right)^{\alpha}\right)^{-2} \times \alpha^{0.1-1}e^{-0.1\alpha} \times \beta^{0.1-1}e^{-0.1\beta} \times \lambda^{0.1-1}e^{-0.1\lambda} \tag{7}$$

The sampling was performed using four parallel chains, each with 20,000 warm-up iterations followed by 10,000 sampling iterations. This yielded a total of 40,000 post-warm-up draws under targeted average acceptance probability of 0.95.
Convergence Diagnostics

Robustness of the MCMC sampling was confirmed with Gelman-Rubin diagnostic ($\hat{R} \approx 1$) for all three parameters. The large Effective Sample Size (ESS > 2600) confirms excellent convergence and reliable posterior distribution (Table 3).

RESULTS AND DISCUSSION

To evaluate the practical utility of the Modified Logistic Lomax (MLL) distribution, we begin by presenting the descriptive characteristics of the data followed by frequentist estimates, which serve as a benchmark for Bayesian results. Subsequently, we report the posterior parameter estimates, convergence diagnostics, model fit criteria, including tools for visual validation. We also report posterior predictive checks as well as survival characteristics. The results consistently indicate that the MLL model, estimated under the Bayesian paradigm, gives good fit to the heavy-tailed data.The summary statistics of the dataset are given in Table 1.

Table 1: Summary Statistics of the dataset.

Min	Q_1	Median	Mean	Q_3	Max	SK	Kurtosis
70	120	133	133.7	146.0	212.0	0.3304	4.0528

For making the study more comprehensive, parameters of the model are also estimated using the maximum likelihood method and are presented in Table 2.

Table 2: Parameter estimates using MLE.

Parameter	Estimate	SE	CI_lower	CI_upper
alpha	8.261	2.305	3.035	11.010
beta	0.137	0.048	0.021	0.185
lambda	0.008	0.004	0.006	0.022

Figure 2 displays the contour plots under maximum likelihood (MLE), least square estimation (LSE) and Cramer-von Mises method (CVM) of estimation.

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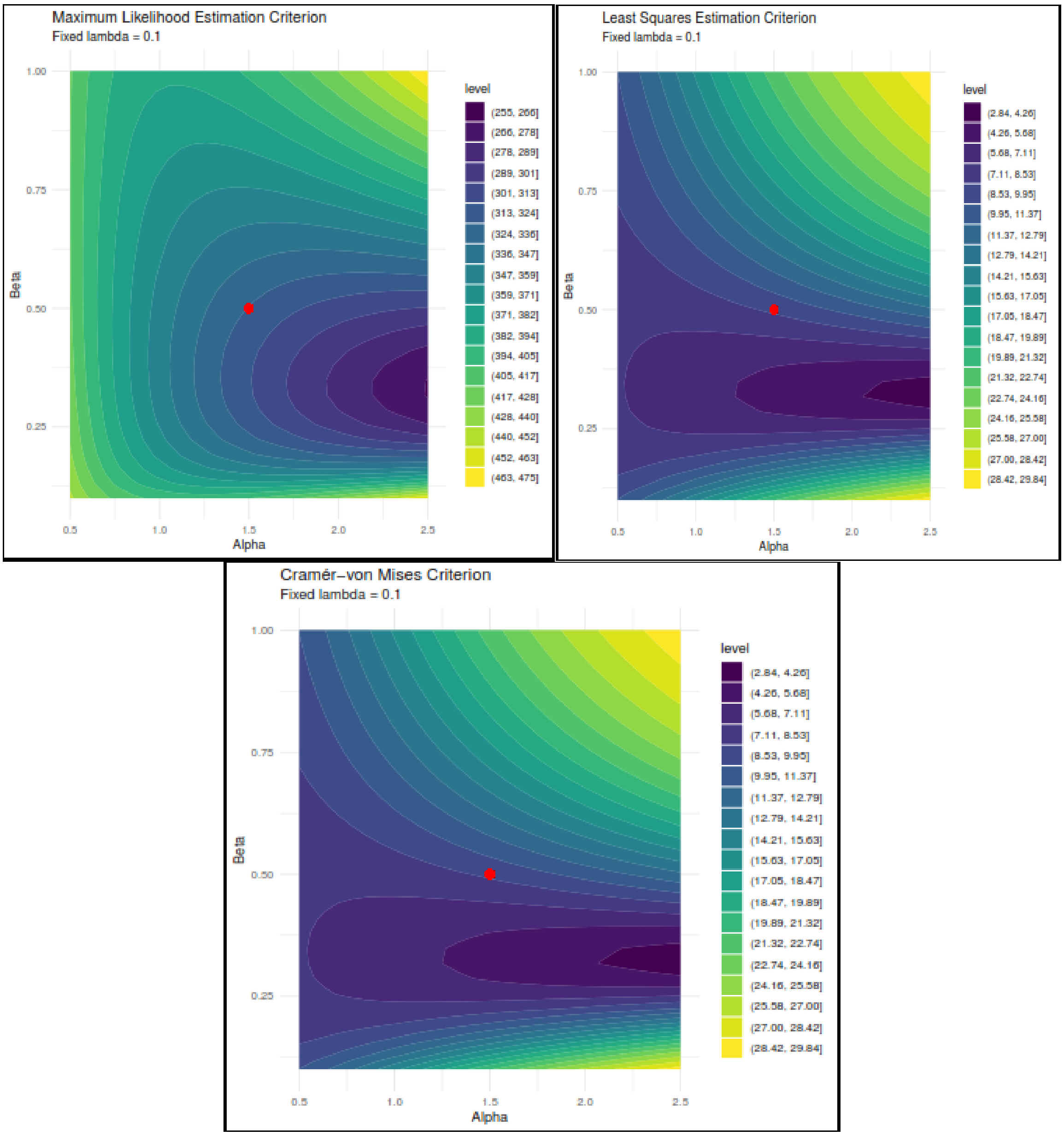

Figure 2: Contour plots under MLE, LSE and CVM.

Posterior Inference

Table 3 also demonstrates the posterior summaries providing a complete probabilistic description of parameter uncertainty. A notable difference is observed between the MLE and Bayesian point estimates, particularly in case of shape parameter α (8.26 vs 3.48). This is the shrinkage effect which is characteristic of Bayesian regularisation, where the Gamma (0.1,0.1) prior pulls the estimate toward the lower values, which effectively reduce the variance, providing more numerically stable inference, especially given the heavy-tailed nature of the data.

Table 3: Posterior summary for the parameters of the model.

Parameter	Mean	Se-mean	sd	2.5%	25%	50%	75%	97.5%	ESS	R-hat
alpha	3.48	0.05	2.32	0.73	1.79	2.87	4.57	9.31	2655	1.01
beta	0.04	0.00	0.04	0.00	0.00	0.02	0.06	0.15	2816	1.00
lambda	0.03	0.00	0.02	0.01	0.01	0.02	0.04	0.09	4876	1.00

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Model Fit and Predictive Performance

The Watanabe-Akaike Information Criterion (WAIC) was used to check the model's predictive accuracy. Estimated effective number of parameter (p-WAIC =2.4) indicates the model is not overfitting (Table 4).

Table 4: WAIC calculations for model comparison.

Metric	Estimate	SE
lpd_WAIC	-458.7	8.9
p_WAIC	2.4	0.6
WAIC	917.4	17.8

Inferential Survival Characteristics

The posterior distributions for key reliability metrics are computed. The Mean Survival Time (MST) is 132.88 (95% CI: 128.52, 137.20) while Median Survival Time (MST50) posterior mean is 133.75 (95% CI: 129.45, 138.10). This allows for direct probability statements about survival times.

Visual Diagnostics of the MLL distribution under Bayesian approach

MCMC Trace Plots

The trace plots for parameters for all three parameters across multiple chains are shown in Figure 3. The trace plots indicate successful convergence.

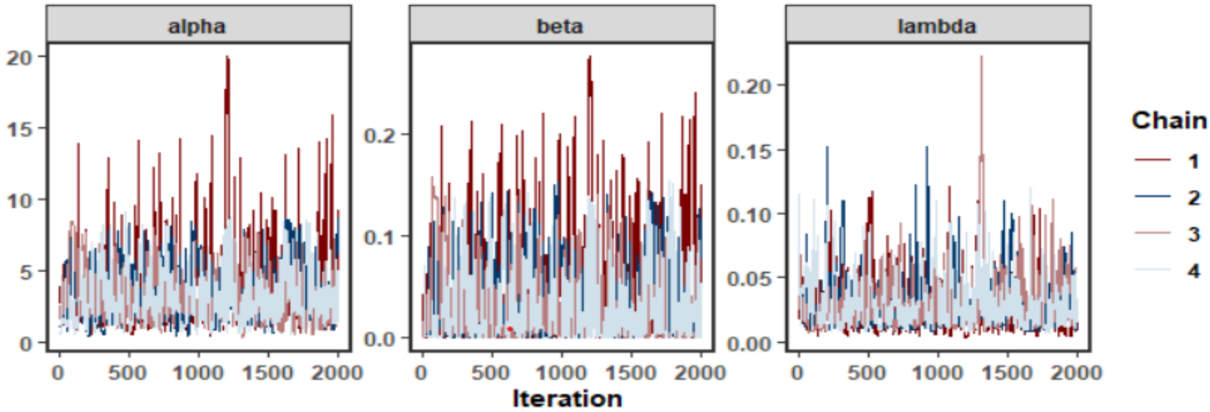

Figure 3: Trace plots of alpha, beta and lambda.

Posterior Distributions of Parameters

Marginal posterior distributions for parameters alpha, beta and lambda are shown in Figure 4.

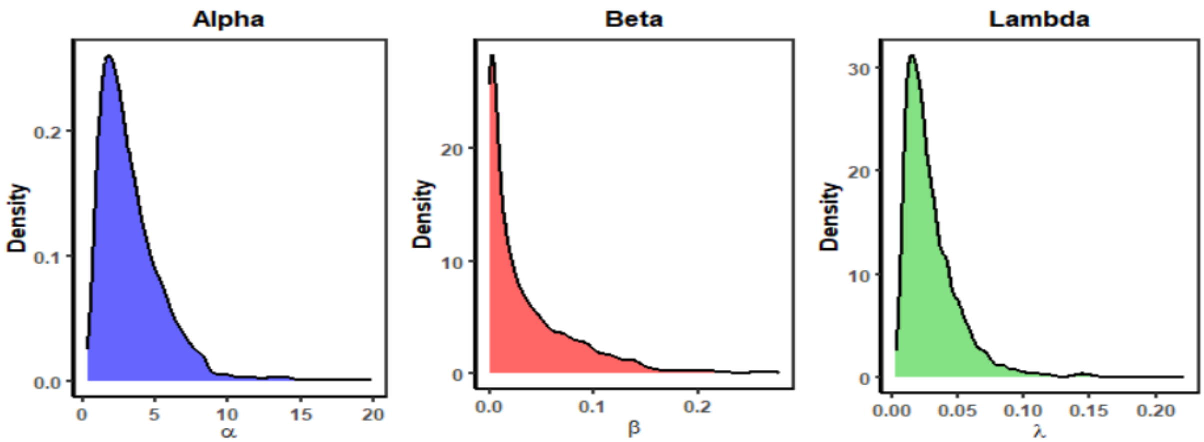

Figure 4: Marginal posterior distributions for alpha, beta, and lambda.

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Posterior Predictive checks

The posterior predictive checks (PPC) for density (left) and cdf (right) are shown in Figure 5. The dark line denotes the observed data aligning closely with 100 simulations from the posterior predictive distribution denoted by light lines, while the overlapping light lines support that the MLL model successfully captures the underlying structure.

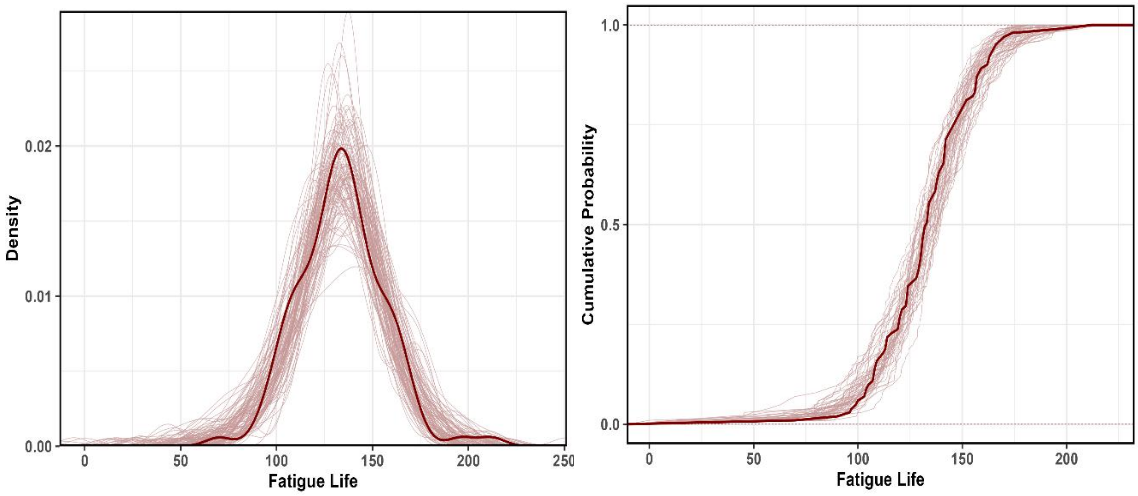

Figure 5: Posterior predictive checks for PDF (left) and CDF (right).

Probability-Probability(P-P) and Quantile -Quantile(Q-Q) plots

P-P and Q-Q plots of the model demonstrated in Figure 6 indicate the MLL model captures the data points more precisely.

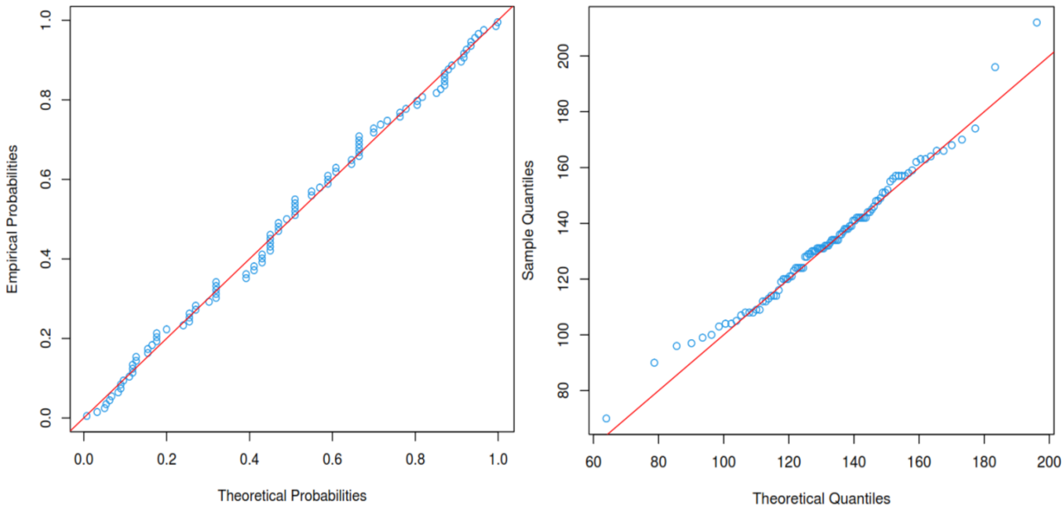

Figure 6: The P-P plot (left) and Q-Q plot (right).

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Posterior Survival plot and Hazard rate plots

The posterior survival plot and posterior hazard rate plots for 95% credible intervals are shown in Figure 7. The narrow 95% credible intervals of the Survival plot confirm high precision in our reliability predictions. To further validate the model's fit, the empirical Kaplan-Meier survival curve was overlaid on the posterior survival plot. The KM curve fell appropriately within the 95% credible intervals across all the points which confirm the MLL model does not deviate systematically from the observed data.

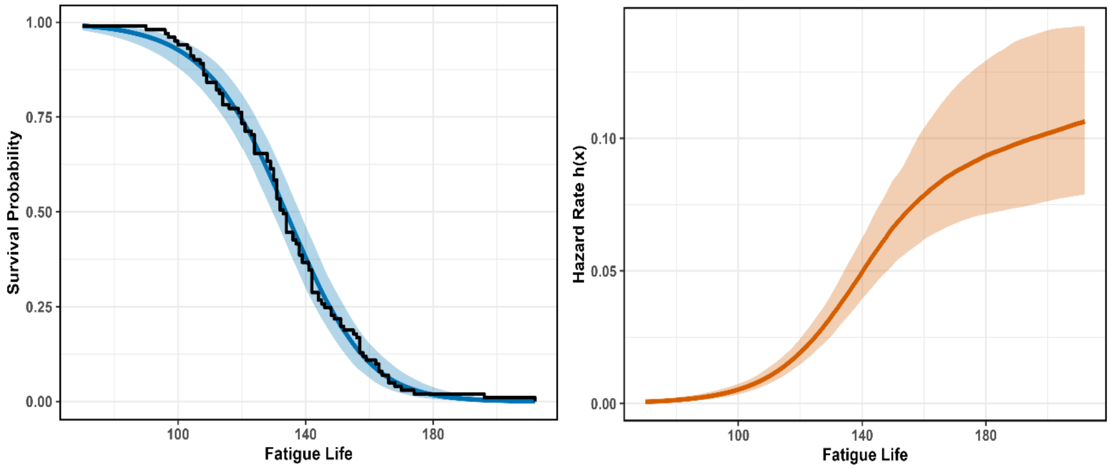

Figure 7: Posterior Survival plot (left) and posterior hazard rate plots (right).

CONCLUSIONS

This study successfully developed a detailed Bayesian framework for analysing heavy-tailed reliability data. Moving beyond traditional likelihood-based methods, we demonstrated the profound advantages of the Bayesian paradigm. Study explores the uncertainty through full posterior distributions, robust parameter estimation confirmed by diagnostics, and direct probabilistic inference for critical reliability metrics, like survival times. This study also confirms that the MLL distribution, estimated within this Bayesian framework, provides a powerful and flexible tool for analysing and modeling complex lifetime data, offering deeper, more intuitive insights for risk assessment and decision-making.

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Ethical approval

There are no any ethical issues with the publication of this article.

Author Contribution

The author is solely responsible for all aspects of this work, including conceptualisation, data analysis, manuscript writing and editing.

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Conflict of Interest

The author declares no conflict of interest.

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