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ARTICLE

Study of the strong coupling constants $\alpha_s(q^2)$ of π^-mesons and protons from p+p interactions at 205 GeV/c

Baatar Tseepeldorj[1*], Batgerel Baatar[1], Alexander Malakhov[2], Khishigbuyan Narankhuu[1], Otgongerel Baatar[1], Sharkhuu Gombojav[1], Sovd Maamuu[1] and Urangua Murdorj[1]

[1] Laboratory of Particle Physics, Department of High Energy Physics, Institute of Physics and Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia
[2] Joint Institute for Nuclear Research, Dubna, Russia

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Abstract: This paper presents an analysis of the strong coupling constant $\alpha_s(q^2)$ for π^-mesons and protons produced in $p+p$ interactions at 205 GeV/c. It is well known that the precise value of the cut parameter Λ_{QCD} is instrumental in obtaining correct values of the strong coupling constant $\alpha_s(q^2)$. In this paper, we have attempted to determine the value of the cut parameter Λ_{QCD} of $\alpha_s(q^2)$. Our analysis showed that the square of the cut parameter Λ_{QCD} of the running coupling constant can be determined by the following formula, $\Lambda^2_{QCD} = (c\hbar)^2\ GeV^2 = 0.0388\ GeV^2$ or $\Lambda_{QCD} = \sqrt{0.0388\ GeV^2} = 0.197\ GeV$. The values of $\alpha_s(q^2)$ with this new cut parameter, as obtained in this paper, are compared with the corresponding LO QCD calculations and resultatively, a good agreement between the experimental results and theory was observed. Thus, the procedure used in this paper provides the possibility of obtaining the exact value of the cut parameter Λ_{QCD} of the running coupling constant $\alpha_s(q^2)$.

Keyword: *high-energy physics, cut parameter, running coupling constant;*

INTRODUCTION

Investigations of the multiparticle production processes in hadron-nucleon (hN) interactions at high energies and large momentum transfers play a very important role in understanding the strong interaction mechanism and the inner quark-gluon structure of nuclear matter [1-4]. In multiparticle production processes at high energies, the secondary particles are produced at different scattering angles with different values of momenta. The scattering angle, particle momentum, and running coupling constant are mainly characterised by the squared four-momentum transfer q^2. In this paper we consider the following reaction:

$$p + p \rightarrow \pi^- + p + X \tag{1}$$

The cut parameter Λ_{QCD} of the running coupling constant was obtained using the following formula:

$$\Lambda^2_{QCD} = (c\hbar)^2 GeV^2 \text{ or } \Lambda_{QCD} = 0.197\ GeV \tag{2}$$

Corresponding author, email: *baatar1945@yahoo.com* *https://orcid.org/0009-0003-9397-7506*

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where c is the speed of light, $\hbar$ is the Planck constant. The choice of this cut parameter considered in this paper is motivated by the general expression for the squared four-momentum transfer q^2. The corresponding dependencies of coupling constants that were calculated using this cut parameter are compared with the leading order (LO) QCD calculations. According to theoretical and experimental studies, the cut parameter is estimated to be between 0.1 and 0.5 GeV [1,4]. This makes it difficult to use the exact value of the cut parameter for the particle produced in the reaction. Therefore, this research aims to obtain an exact value of the cut parameter for the particles produced in the considered reactions.

MATERIALS AND METHODS

Data on $p+p$ interactions at 205GeV/c were obtained with the help of a 30-inch hydrogen bubble chamber at NAL (National Accelerator Laboratory). The 5025 $p+p$ events containing 12095 π^-mesons and 2086 protons at 205GeV/c were used in the analysis.

The cumulative number n_c. The variable n_c, called the cumulative number in the fixed target experiment, is determined by the following formula [3,5]:

$$n_c = \frac{P_a \cdot P_c}{P_a \cdot P_b} = \frac{E_c - \beta_a P_c^{||}}{m_p} \tag{3}$$

where P_a, P_b and P_c are the four-dimensional momenta of the incident, target and secondary particles under consideration. E_c and $P_c^{||}$ are the energy and longitudinal momentum of the secondary particle, respectively, and $\beta_a = P_a/E_a$ is the velocity of the incident particle. At high energy experiments $\beta_a{\sim}1$ and m_p is the proton mass. It can be seen from formula (3) that variable n_c is a relativistic invariant and dimensionless. Furthermore, this variable makes it possible to determine which particles in the event under consideration, are produced in the cumulative region $(n_c > 1)$ and vice versa. Figures 1 and 2 show the n_c distributions of π^- mesons and protons from reaction (1). When calculating the value of q^2 by formula (4) for the reaction $p+p \to \pi^- + p + X$ interaction at 205 GeV/c the secondary energetic protons with small n_c produce particles with negative values of q^2. These particles were excluded from the physical distribution because they correspond to unphysical values of q^2(see Table 2).

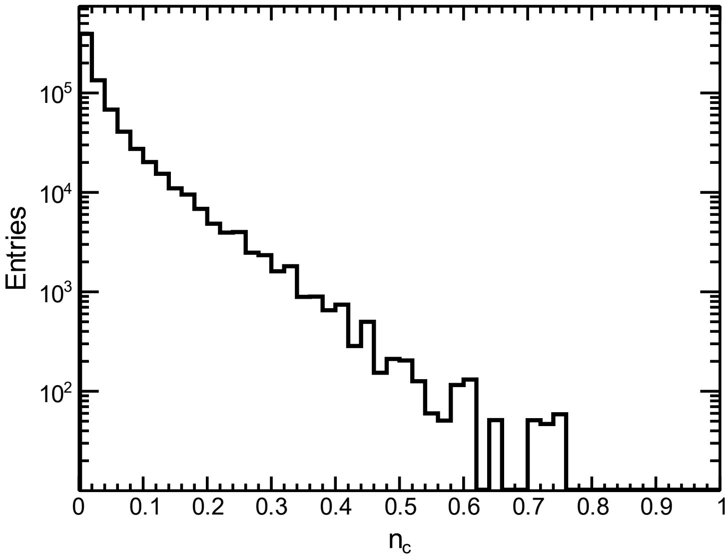

Figure 1. n_c distribution of π^- mesons from $p+p \to \pi^- + X$ interactions at 205 GeV/c.

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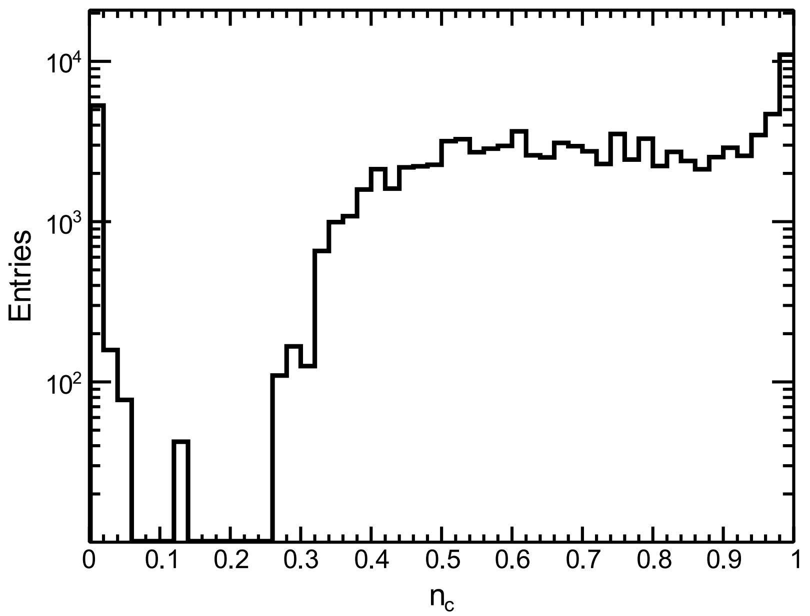

Figure 2. n_c **distribution of protons from** $p + p \rightarrow p + X$ **interactions at 205 GeV/** c**.**

RESULTS AND DISCUSSION

The squared four-momentum transfer q^2 **and the running coupling constant** $\alpha_s(q^2)$**.** As mentioned in the Introduction, the squared four-momentum transfer q^2 plays an important role in the description of multiparticle production processes at high energies [1,4]. In the present analysis, q^2 was calculated for secondary π^- mesons and protons produced in $p + p$ interactions at 205 GeV/c. The squared four-momentum transfer is determined by the following formula:

$$\begin{aligned} q^2 &= -(P_a - P_c)^2 = 2E_a\left(E_c - \beta_a P_c^{||}\right) - (m_a^2 + m_c^2) \\ &= 2E_a m_p n_c - (m_a^2 + m_c^2) \end{aligned} \tag{4}$$

where E_a and m_a are the energy and mass of the incident particle, respectively, mc is the mass of the secondary particle under consideration, and the other notations are the same as defined above. Formula (4) can be rewritten in the following form using formula (3),:

$$\frac{q^2}{m_a^2 + m_c^2} = \frac{2E_a m_p n_c - (m_a^2 + m_c^2)}{m_a^2 + m_c^2} = \frac{2E_a m_p n_c}{m_a^2 + m_c^2} - 1. \tag{5}$$

Taking the natural logarithm of both sides gives

$$\ln\left(\frac{q^2}{m_a^2 + m_c^2}\right) = \ln\left(\frac{2E_a m_p n_c - (m_a^2 + m_c^2)}{m_a^2 + m_c^2}\right) = \ln\left(\frac{2E_a m_p n_c}{m_a^2 + m_c^2} - 1\right). \tag{6}$$

On the other hand, in the leading-order (LO) approximation, the strong coupling constant as a function of the squared four-momentum transfer q^2 is expressed as [4,6] follows:

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$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2}{\Lambda^2}\right)} = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2}{m_a^2+m_c^2}\right)}. \tag{7}$$

To determine the scale parameter used in formula (7), the mass term in formula (4) was written through the corresponding Compton wavelengths of the incident and secondary particles. Using $mi^2 = (c\hbar)^2/(\lambda C^i)^2$ one obtains:

$$m_a^2 + m_c^2 = \frac{(c\hbar)^2 GeV^2 fm^2}{(\lambda_C^a)^2 fm^2} + \frac{(c\hbar)^2 GeV^2 fm^2}{(\lambda_C^c)^2 fm^2} = (c\hbar)^2 \left[\frac{(\lambda_C^a)^2 + (\lambda_C^c)^2}{(\lambda_C^a)^2(\lambda_C^c)^2}\right] = k_{ac}(c\hbar)^2 GeV^2 \tag{8}$$

where k_{ac} is the dimensionless parameter. In this analysis, the parameter k_{ac} plays the role of a gauge parameter. From formula (8), one obtains

$$(c\hbar)^2 = \frac{m_a^2 + m_c^2}{k_{ac}} \tag{9}$$

A change in the renormalisation point can be compensated by a corresponding change in the gauge parameter. As a result, the universal constant obtained in formula (9) can be used as the cut parameter of the running coupling constant. This is consistent with the theoretical statement given in Ref. [6], where the choice of the renormalisation point μ is arbitrary, and any change in μ can be compensated by corresponding changes in the charge, the scale of the fields, and the gauge parameter. Thus, the cut parameter of the running coupling constant is determined as

$$(c\hbar)^2 = \frac{m_a^2 + m_c^2}{k_{ac}} \tag{10}$$

and finally

$$\Lambda_{QCD}^2 = (c\hbar)^2 GeV^2 = (0.197)^2 GeV^2 = 0.0388 GeV^2 \tag{11}$$

or

$$\Lambda_{QCD} = 0.197\ GeV.$$

The experimental dependences of $\boldsymbol{\alpha_s(q^2)}$. The strong coupling constant in the LO approximation was calculated using the following expression:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{\Lambda^2}\right)}, \qquad \beta_0 = 11 - \frac{2}{3}n_f \tag{12}$$

where n_f is the number of active quark flavours. Figures 3 and 4 show the dependences of $\alpha_s(q^2)$for π^- mesons and protons produced in reaction (1), respectively. The data points represent the experimental results obtained from the present analysis, while the curves represent the corresponding LO QCD calculations.

The values of the cut parameter Λ_{QCD} and the number of flavours n_f were obtained using the Fumili minimisation procedure. The fitted results are shown directly in Figures 3 and 4. For Figure 3, the Fumili fit was performed for the π^--meson data from Table 1, excluding the first low-q^2 point from the fit region, which gives

$\chi^2/ndf = 5.839/9$. The main characteristics of the π^--meson and proton samples are presented in Tables 1 and 2 respectively.

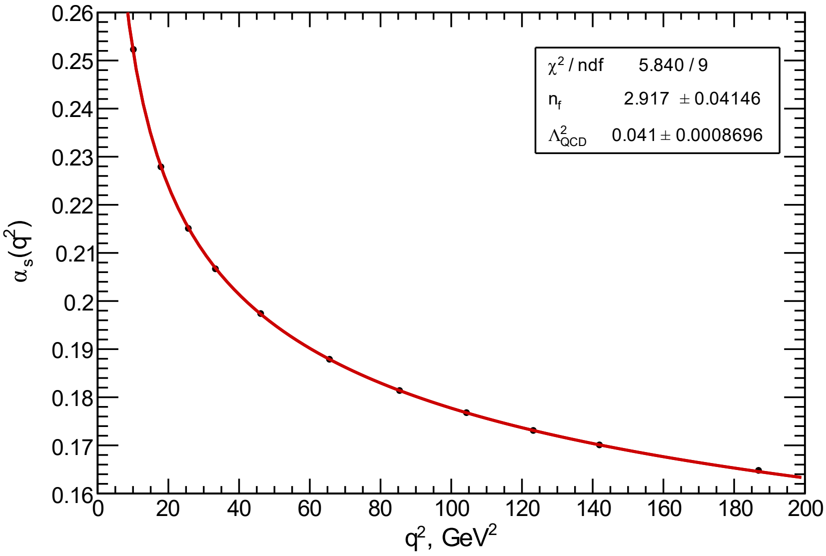

Figure 3. Dependence of $\alpha_s(q^2)$ for π^- mesons from $p+p \to \pi^- + X$ interactions at 205 GeV/.

Table 1. Characteristics of π^- mesons from $p+p \to \pi^- + X$ interactions at 205 GeV/c.

№	N_{π^-}	n_c	$\langle n_c \rangle$	$q^2(GeV^2)$	$\alpha_s(q^2)$
1	6327	0.0 – 0.02	0.00821 ± 0.00007	2.637 ± 0.02597	0.4954 ± 0.02561
2	2098	0.02 – 0.04	0.02861 ± 0.0001	10.11 ± 0.04816	0.2523 ± 0.00021
3	1075	0.04 – 0.06	0.04882 ± 0.0002	17.89 ± 0.06703	0.2279 ± 0.00013
4	653	0.06 – 0.08	0.0687 ± 0.0002	25.64 ± 0.08432	0.2151 ± 0.00010
5	440	0.08 – 0.1	0.0889 ± 0.0003	33.3 ± 0.1054	0.2067 ± 0.00009
6	695	0.1 – 0.15	0.1221 ± 0.0005	46.1 ± 0.2112	0.1974 ± 0.00012
7	348	0.15 – 0.2	0.1726 ± 0.0008	65.56 ± 0.3012	0.1879 ± 0.00012
8	186	0.2 – 0.25	0.2246 ± 0.001	85.38 ± 0.4299	0.1814 ± 0.00012
9	114	0.25 – 0.3	0.2729 ± 0.001	104.3 ± 0.5291	0.1768 ± 0.00011
10	69	0.3 – 0.35	0.3225 ± 0.001	123.2 ± 0.5899	0.1731 ± 0.00010
11	38	0.35 – 0.4	0.3714 ± 0.002	141.9 ± 0.8456	0.1701 ± 0.00010
12	50	0.4 – 1.0	0.4872 ± 0.01	186.9 ± 4.729	0.1648 ± 0.00012

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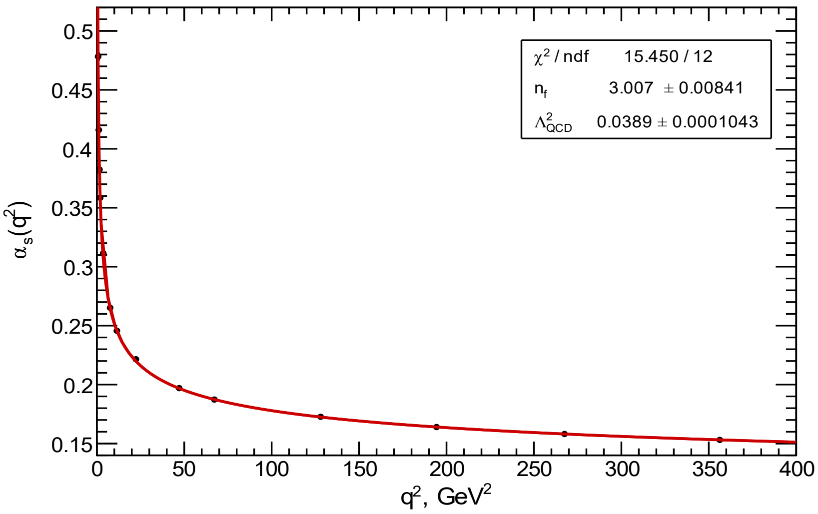

Figure 4. Dependence of $\alpha_s(q^2)$ for protons from $p + p \to p + X$ interactions at 205 GeV/c.

Table 2. Characteristics of protons from $p + p \to p + X$ interactions at 205 GeV/c.

№	N_p	n_c	$\langle n_c \rangle$	$q^2(GeV^2)$	$\alpha_s(q^2)$
1	1603	0.0 – 0.005	0.002629 ± 0.00001	-0.7494 ± 0.01153	2.018 ± 0.9316
2	245	0.005 – 0.006	0.005479 ± 0.0004	0.3469 ± 0.006956	0.6686 ± 0.0007
3	221	0.006 – 0.007	0.006488 ± 0.0004	0.7345 ± 0.008	0.4783 ± 0.0017
4	165	0.007 – 0.008	0.007493 ± 0.00056	1.122 ± 0.0087	0.416 ± 0.0009
5	110	0.008 – 0.009	0.008473 ± 0.000067	1.499 ± 0.01011	0.3825 ± 0.00006
6	100	0.009 – 0.01	0.009544 ± 0.000026	1.911 ± 0.0098	0.3585 ± 0.0004
7	532	0.01 – 0.02	0.01444 ± 0.00013	3.678 ± 0.05027	0.3114 ± 0.0009
8	216	0.02 – 0.03	0.02457 ± 0.0002	7.614 ± 0.0794	0.2652 ± 0.0005
9	156	0.03 – 0.04	0.03436 ± 0.00023	11.46 ± 0.08558	0.2457 ± 0.0003
10	285	0.04 – 0.10	0.06267 ± 0.001	22.35 ± 0.3847	0.2215 ± 0.0005
11	95	0.10 – 0.16	0.127 ± 0.0018	47.09 ± 0.7098	0.197 ± 0.0004
12	34	0.16 – 0.2	0.1794 ± 0.0020	67.25 ± 0.7749	0.1873 ± 0.0003
13	130	0.2 – 0.4	0.3377 ± 0.0049	128 ± 1.855	0.1727 ± 0.0003
14	465	0.4 – 0.6	0.5097 ± 0.0029	194.3 ± 1.003	0.164 ± 0.00009
15	565	0.6 – 0.8	0.6999 ± 0.0025	267.5 ± 0.9656	0.1581 ± 0.00006
16	761	0.8 – 1.0	0.9307 ± 0.0022	356.3 ± 0.8527	0.1531 ± 0.00005

The obtained experimental values of $\alpha_s(q^2)$ decrease by increasing q^2, which is consistent with the expected behaviour of the running coupling constant in QCD. The fitted LO QCD curves describe the satisfactory nature of the experimental data within the considered q^2 range.

This agreement indicates that the selected value of Λ_{QCD} provides a reasonable description of the experimental $\alpha_s(q^2)$ dependences for secondary π^- mesons and protons produced in $p + p$ interactions at 205 GeV/c.

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Figures 5-7 present a comparison of the present results with the world measurements of the running coupling constant α_s as a function of the energy scale (q) [7]. The QCD curves represent the world average prediction of the running coupling constant, while the present results obtained from reaction (1) are superimposed on the corresponding world data.

At comparatively small transferred momentum, where experimental measurements are limited, the present results provide additional data points. A good agreement is observed between the present experimental results and the world measurements over the investigated energy range.

All experimental points from Table 1 are shown in the world-data comparison, while the first low-q^2 point was excluded only from the Fumili fit shown in Figure 3.

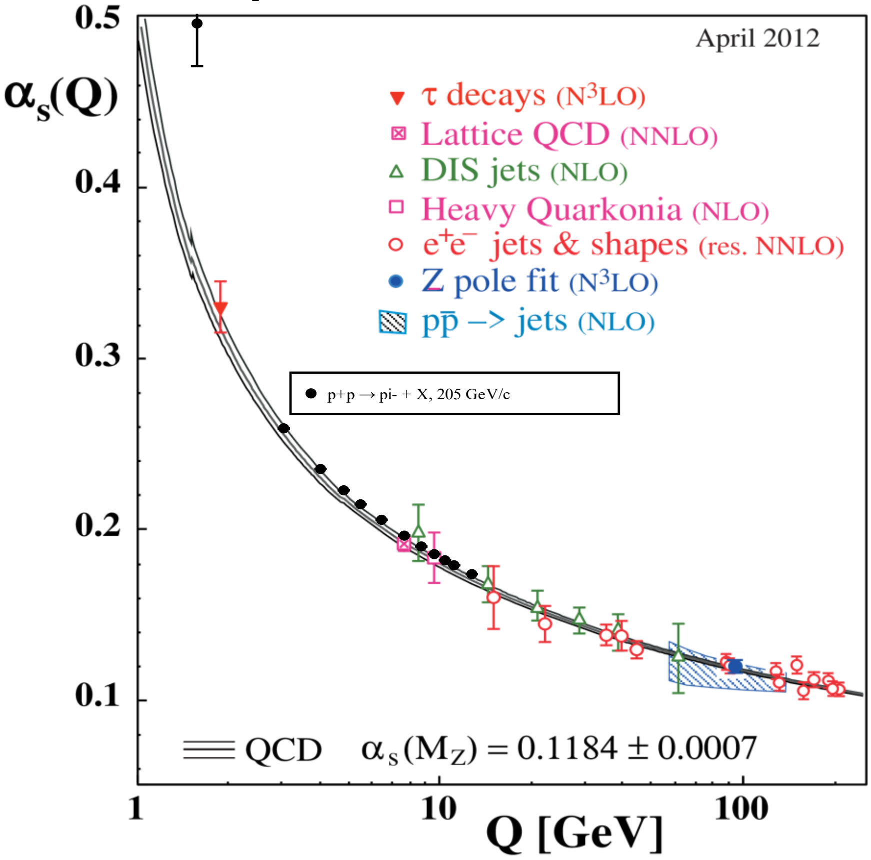

Figure 5. Summary of measurements of α_s as a function of the energy scale q for π^- mesons from $p + p \to \pi^- + X$ interactions at 205GeV/c. The QCD curves represent the world-average prediction, while the present result is shown together with the experimental data.

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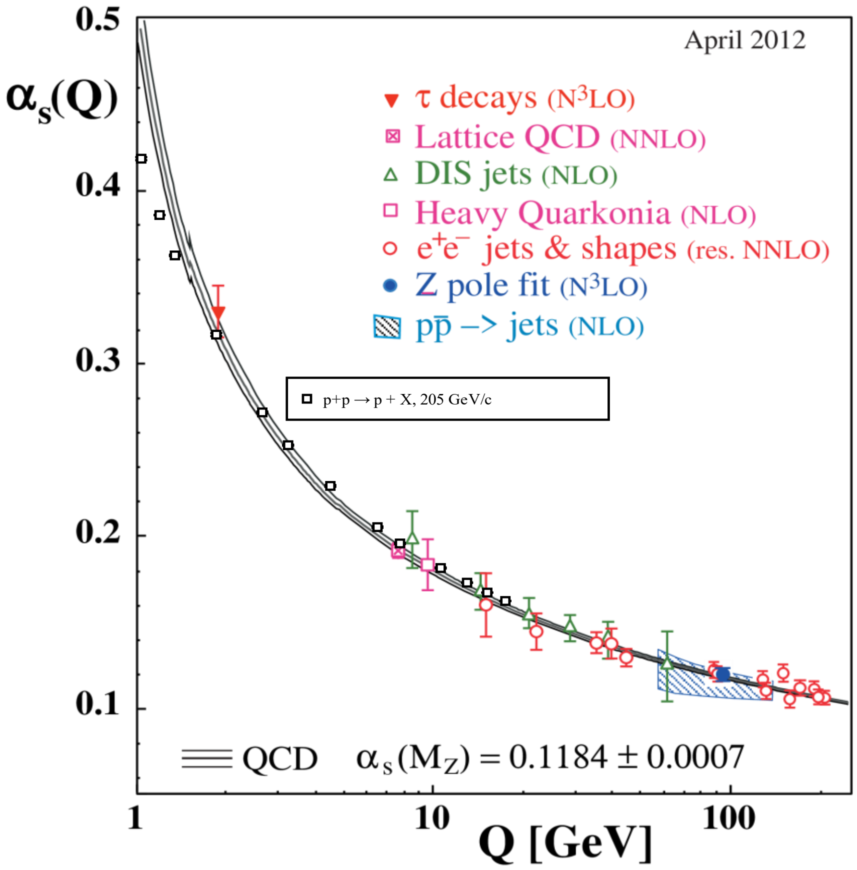

Figure 6. Summary of measurements of α_s as a function of the energy scale q for protons from $p+p \to p+X$interactions at 205GeV/c. The QCD curves represent the world-average prediction, while the present result is shown together with the experimental data.

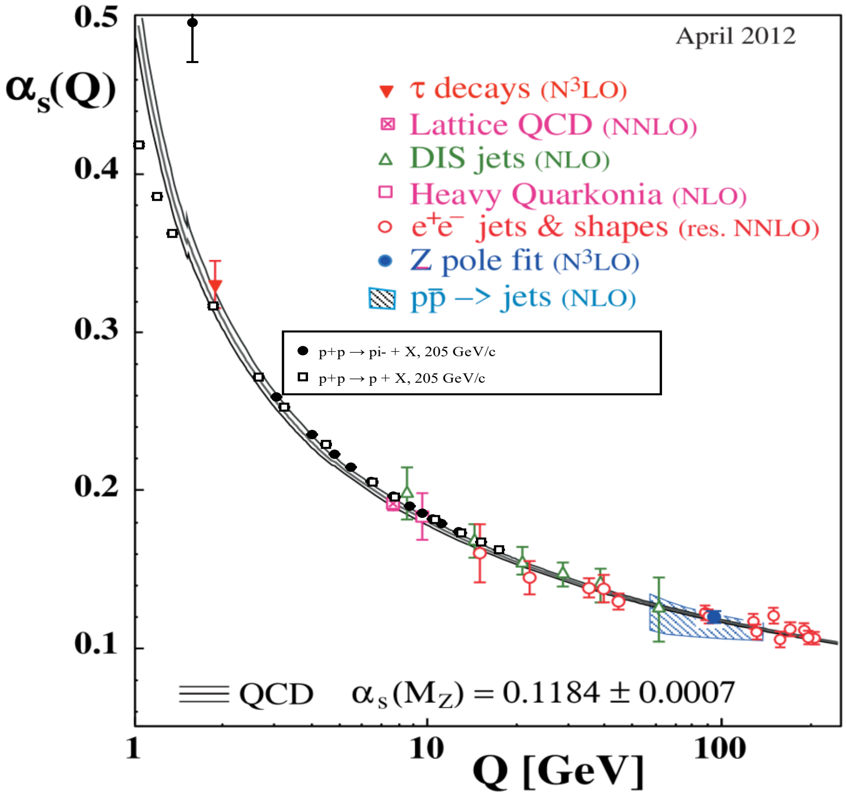

Figure 7. Combined comparison of the present α_s values for π^-mesons and protons from $p+p$ interactions at 205 GeV/c with world measurements of α_s as a function of the energy scale (q).

CONCLUSIONS

In this work, the running coupling constant $\alpha_s(q^2)$ for π^- mesons and protons produced in $p+p$ interactions at 205 GeV/c was analysed. As a result of the analysis, the cut parameter of the running coupling constant was determined as follows:

$$\Lambda_{QCD}^2 = (c\hbar)^2 GeV^2 = (0.197)^2 GeV^2 = 0.0388 GeV^2,$$

or

$$\Lambda_{QCD} = 0.197\ GeV.$$

The obtained values of $\alpha_s(q^2)$, calculated using this cut parameter, were compared with the corresponding LO QCD calculations. At comparatively small q^2, where experimental measurements are limited, the obtained experimental data are in good agreement with the QCD calculations. The results are also consistent with other experimental data obtained at higher q^2 values. Therefore, the present analysis supports the applicability of the selected cut parameter (Λ_{QCD}) in the calculation of the running coupling constant $\alpha_s(q^2)$ for secondary particles produced in $p+p$ interactions at 205 GeV/c.

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Author contribution

The authors confirm contribution to the paper as follows: Ts.B: study conception and design, methodology, data analysis, calculations, interpretation of results, manuscript preparation, and overall coordination of the research; B.O: manuscript review, editing, and preparation of the revised version; A.I.M: scientific supervision, consultation, and review of the manuscript; B.B, N.Kh, G.Sh, M.S and M.U: support in data checking, discussion of results, editing, and manuscript review. All authors read and approved the final version of the manuscript.

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Conflict of interest

The authors declare that they have no conflict of unterest.

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