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ARTICLE

Study of the strong coupling constants, $\alpha_s(q^2)$ of π^- and K^0 mesons from $\pi^- + p$ and $\pi^- + C$ interactions at 40 GeV/c

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Abstract: This paper is devoted to the production of $\pi^-$and K^0 mesons from $\pi^- + p \rightarrow \pi^- + X$ and $\pi^- + C \rightarrow \pi^-, K^0 + X$ interactions at 40 GeV/c as a function of the square of four momentum transfer. The cut parameter of the strong coupling constant is taken as $\Lambda_{ac}^2 = m_a^2 + m_c^2$. Values of the strong coupling constant are then compared to leading-order and next-to-leading-order perturbative QCD calculations for the first time. Agreement between the experimental data and theory is good, thus providing a precision test of QCD at large momentum transfers (q). The strong coupling constant α_s is extracted as a function of q, showing a good agreement with the renormalisation group equation and with previous analyses. As the momentum transfers increases, the running coupling constant decreases. For each high-energy interaction, a quantity called the cut parameter is chosen differently depending on the secondary particles produced by the reaction. For each high-energy interaction, a quantity called the cut parameter is chosen differently depending on the secondary particles produced by the reaction.

INTRODUCTION

Investigations of the multiparticle production processes in hadron-nucleon (hN), hadron-nucleus (hA) and nucleus-nucleus (AA) interactions at high energies and large momentum transfers, play a very important role in understanding the strong interaction mechanism and the inner quark-gluon structure of nuclear matter. It is well known that in hA and AA interactions at high energies secondary particles are produced in the region kinematically forbidden for hN interactions.

In other words, the cumulative number of particles in hA and AA interactions at high energies are produced at large momentum transfers, not allowed for hN interactions. Thus, studying the cumulative particle production process allows for examination of the features of the nuclear matter under extreme conditions. Under these conditions (multiparticle production processes at high energies), the secondary particles are produced at different scattering angles with different values of momenta.

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Values of the scattering angle, momentum and running coupling constants are mainly described by square of the four-dimensional transferred momentum q^2 .

This paper focuses on the study of the π^- and K^0 mesons running coupling constant, determined from the following reactions run at 40 GeV/c:

$$\pi^- + p \rightarrow \pi^- + X \qquad \text{(Reaction 1)}$$

$$\pi^- + C \rightarrow \pi^-, K^0 + X \qquad \text{(Reaction 2)}$$

In this paper, the cut parameter Λ of the running coupling constant was chosen, based on by the following formula:

$$\Lambda_{ac}^2 = m_a^2 + m_c^2 \qquad (1)$$

where m_a is the mass of the incident particle and m_c is the mass of the secondary particle under consideration.The choice of this cut parameter is considered in this paper, as are the corresponding dependences of coupling constants obtained using this cut parameter, for the LO (leading order) and NLO (next-to leading order) cases. According to theoretical calculations [1], the cut parameter is estimated to be between 0.1-0.5. This makes it difficult to use the exact value of the cut parameter for the particle involved in the reaction. Therefore, this research aims to derive a cut parameter for an exact value for the particles involved in such a reaction.

MATERIALS AND METHODS

Experimental method

The experimental material was obtained using the Dubna 2-meter propane (C_3H_8) bubble chamber exposed to π^-mesons with a momentum of 40 GeV/c from the Serpukhov accelerator. The advantage of the bubble chamber experiment in this paper is that the distributions are obtained under the condition of 4π geometry of secondary particles. The average error of the momentum measurements is ~12% and the average error of the angular measurements is ~0.6%.

The 8791 $\pi^- + C$ events with 30162 π^-mesons and 11576 $\pi^- + p$events with 31224 π^-mesons at 40 GeV/c have been used in the experimental analysis. For the analysis of π^-, K^0 combination 5800 $\pi^- C$ interactions with the detection of neutral particles have been used. The 554 K^0 mesons were detected in these interactions and used in this analysis.

The cumulative number $\mathbf{n_c}$

The variable n_c , called the cumulative number in the fixed target experiment, is determined by the following formula [2,3];

$$n_c = \frac{P_a P_c}{P_a P_b} = \frac{E_c - \beta_a P_c^{||}}{m_p} \qquad (2)$$

where P_a, P_b and P_c are the four-dimensional momenta of the incident, target and secondary particles under consideration. E_c and $P_c^{||}$ are the energy and longitudinal momentum of the secondary particle, respectively, and $\beta_a = \frac{P_a}{E_a}$ is the velocity of the incident particle. At high energy experiments $\beta_a = 1$ and m_p is the proton mass.

It can be seen from formula (2) that the variable n_c is a relativistic invariant and

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dimensionless. Furthermore, this variable provides an opportunity to know which particles in the event under consideration, are produced in the cumulative region ($n_c > 1$) and vice versa.

Figure 1 a, b shows n_c distributions of π^- mesons from $\pi^- + p$ and $\pi^- + C$ interactions at 40 GeV/c. It should be noted that the n_c distribution of π^--mesons from $\pi^- + p$ interaction continues until 1 and the corresponding distribution from $\pi^- + C$ interaction continues until ~5. The difference between these two interactions is connected to the production of cumulative π^- mesons in $\pi^- + C$ interaction. Figure 1c shows the n_c distribution of K^0-mesons from the $\pi^- + C$ interaction at 40 GeV/c.

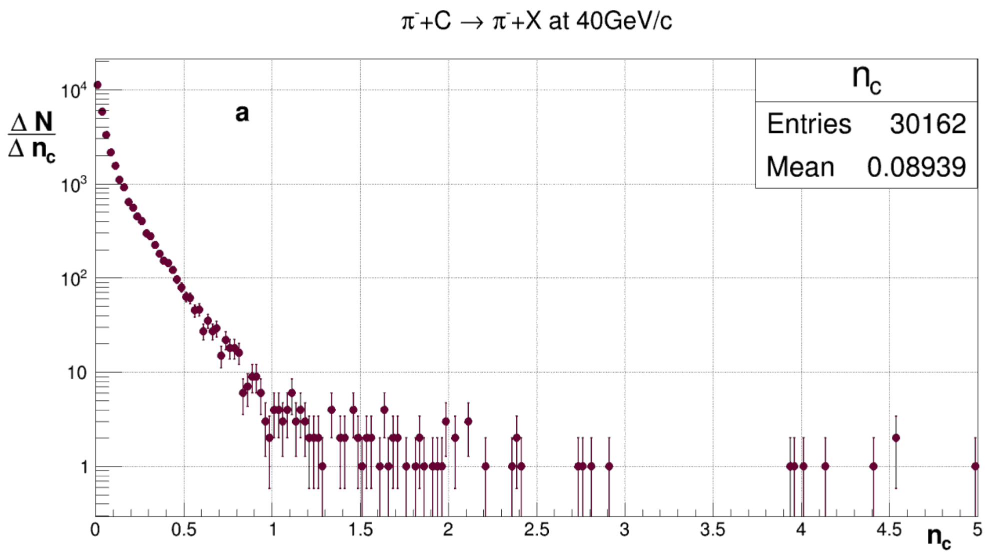

Figure 1a.The n_c distribution of π^- mesons

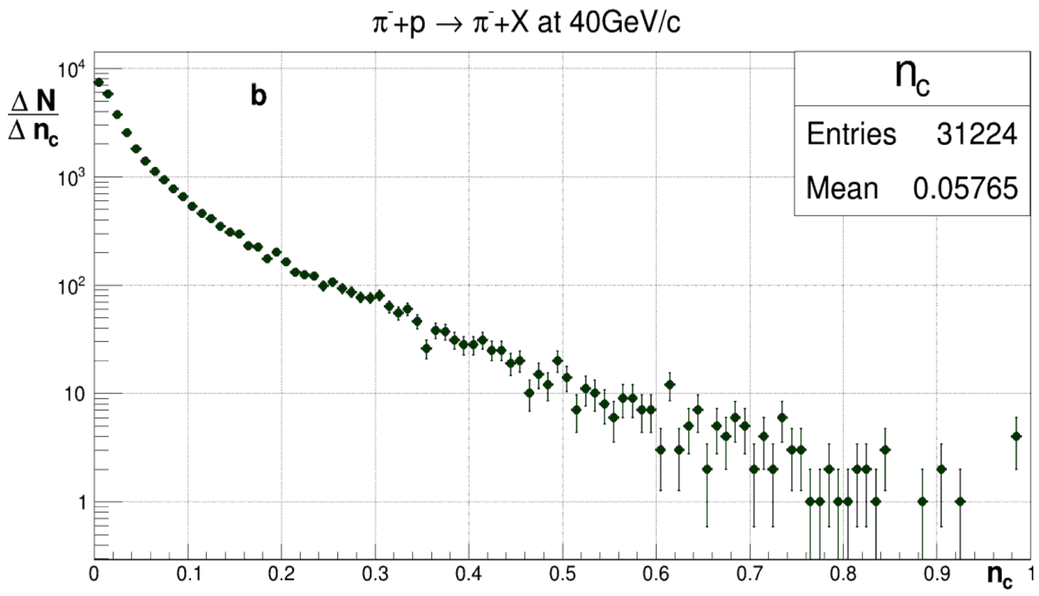

Figure 1b.The n_c distribution of π^- mesons

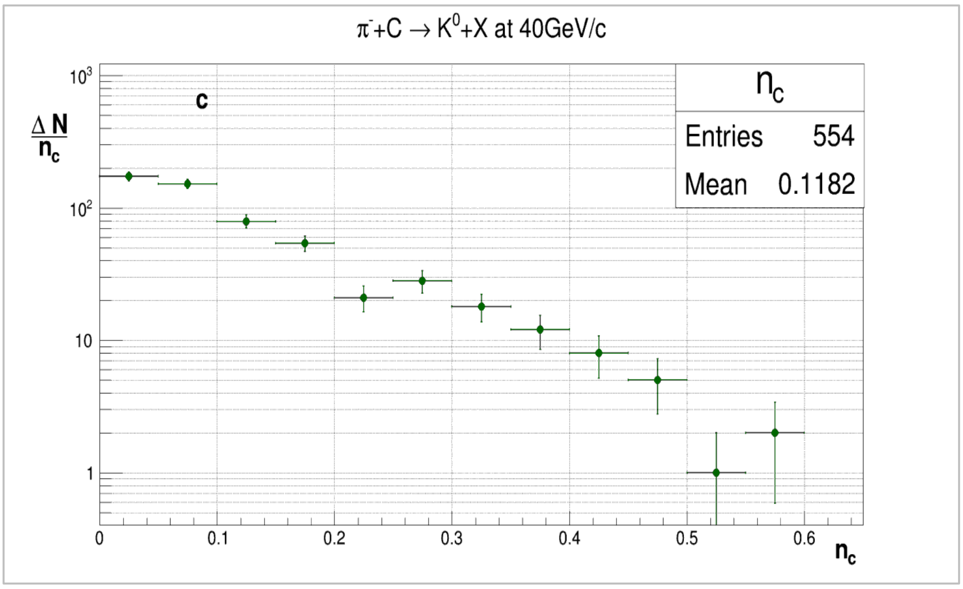

Figure 1c.The n_c distribution of K^0 meson

The square of 4-momentum transfer q^2 and the running coupling constant

As mentioned in the Introduction, the square of the momentum transfer q^2 plays very important role in the multiparticle production process at high energies. The square of the transferred momentum q^2 is determined by the following formula:

$$q^2 = -(P_a - P_c)^2 = 2E_a\left(E_c - \beta_a P_c^{||}\right) - (m_a^2 + m_c^2) \tag{3}$$

where E_a and m_a are the energy and mass of the incident energy particle, respectively m_c is the mass of the secondary particles under consideration, and the other notations are the same as in Subsection 3. Formula 3 may be rewritten in the following form using Formula 2;

$$\frac{q^2}{m_a^2+m_c^2} = \frac{2E_a m_p n_c - (m_a^2+m_c^2)}{m_a^2+m_c^2} = \frac{E_a m_p n_c}{m_a^2+m_c^2} - 1 \tag{4}$$

Now we can take the natural logarithm from both sides of this formula,

$$ln\left(\frac{q^2}{m_a^2+m_c^2}\right) = ln\left(\frac{2E_a m_p n_c - (m_a^2+m_c^2)}{m_a^2+m_c^2}\right) = ln\left(\frac{E_a m_p n_c}{m_a^2+m_c^2} - 1\right) \tag{5}$$

In the first term of Equation (5), the coupling constant of the strong interaction in the LO approximation as a function of the 4-momentum transfer q^2 is determined by the following Formula:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{\Lambda^2}\right)} = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{m_a^2+m_c^2}\right)} \tag{6}$$

Formula (6) gives us the possibility to obtain the following identity $\Lambda_{ac}^2 = m_a^2 + m_c^2$ (formula 1). This means that the unknown cut parameter Λ^2 in the multiparticle production processes

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can be expressed as the sum of the squares of masses of the

incident and investigating secondary particles, in other words, the cut parameter of the strong coupling constant Λ^2 essentially depends on the different combinations of masses of the incident and investigated secondary particles. With this goal, the different reactions of the multiparticle production processes can be considered.

If the reaction in which the incident and the investigated secondary particle under consideration are believed to be the same then for example,

$$\pi^- + p(C) \to \pi^- + X$$

In this case, according to formula (1)

$$= 2m_\pi^2 = 0.038462\ GeV^2 \text{ or } \Lambda_{\pi\pi} = \sqrt{2}m_\pi = 0.197 \approx\ GeV \qquad (7)$$

Considering the reaction, $p + p \to p + X$,then the following equations are obtained:

$$\Lambda_{pp}^2 = 2m_p^2 \text{ or } \Lambda_{pp} = \sqrt{2}m_p m_p = \frac{\Lambda_{pp}}{\sqrt{2}} \qquad (8)$$

Considering the reaction in which the incident and the investigated particles are different, for example,

$$\pi^- + C \to K^0 + X$$

In this case Formula (3) yields the following formula:

$$\Lambda_{\pi K^0}^2 = m_\pi^2 + m_{K^0}^2 \text{ or } \Lambda_{\pi K^0}^2 = 2\left(\frac{m_\pi^2 + m_{K^0}^2}{2}\right) = 2\langle m_{\pi K^0}^2\rangle \text{ or} \Lambda_{\pi K^0} = \sqrt{2}\langle m_{\pi K^0}\rangle = 0.516\ GeV \qquad (9)$$

The experimental values of the cut parameter Λ obtained from the different experiments are the following [2];

$$\Lambda \simeq (0.1 \div 0.5)GeV$$

The usually quoted value of the parameter is $\Lambda \simeq 0.2$GeV, but it is not well determined and the data from the HERA experimental collider suggest a value $\Lambda \simeq 0.3$GeV [3].

It should be stressed that the choice of Formula (1) in this paper for reactions $\pi^- + p \to \pi^- + X$ and $\pi^- + C \to \pi^- + X$ gives the same value $\Lambda \simeq 0.2$GeV.

It is interesting to note that even though the above two, different reactions, with the same and different incident and investigating secondary particles, in both cases, they obey the linear equation with the same slope parameter ($\tan(\alpha) = \frac{1}{\sqrt{2}} = 0.7071\ or\ \alpha = 32.26^0$). This dependence is shown in Figure 2.

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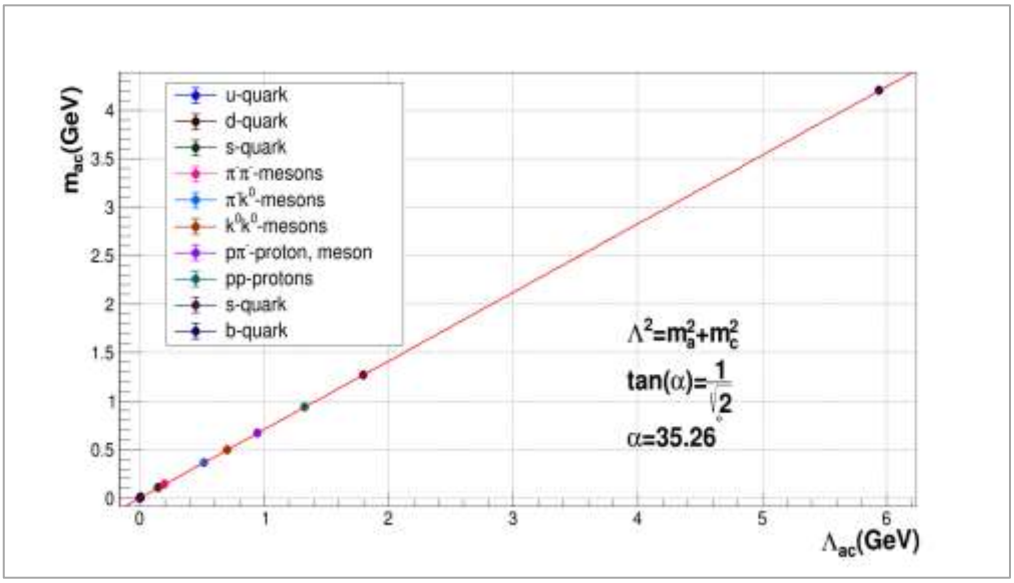

Figure 2. Dependence of the mass m_{ac} on Λ_{ac}

The variable $\alpha_s(q^2)$ with radiation correction

The strong coupling constant with radiation corrections (NLO) is determined by Formula10, which follows, based on previously published [4,5]:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{\Lambda^2}\right)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{ln\left(ln\left(\frac{q^2}{\Lambda^2}\right)\right)}{ln\left(\frac{q^2}{\Lambda^2}\right)}\right] \quad (10)$$

$$\beta_0 = 11 - \frac{2}{3}n_f,$$

$$\beta_1 = 51 - \frac{19}{3}n_f,$$

where β_0 and β_1 are the coefficients obtained when calculating the different radiation corrections. Figures 3 a to c show $\alpha_s(q^2)$ dependence of π^-, K^0 mesons on Reaction 1 and Reaction 2 for LO and NLO. The data points are the experimental results. The red and blue lines correspond to the fitting process we obtained using $\Lambda = 0.197$ GeV for $\pi^-\pi^-$ combination and $\Lambda = 0.516$ GeV for π^-K^0 combination, respectively. These results are within the limits of the experimental accuracy (0.1-0.5). The corresponding values of χ^2 are presented in Panels a-c in Figure 3. The first term of Formula 10 determines the LO approximation and the second term corresponds to NLO approximation.

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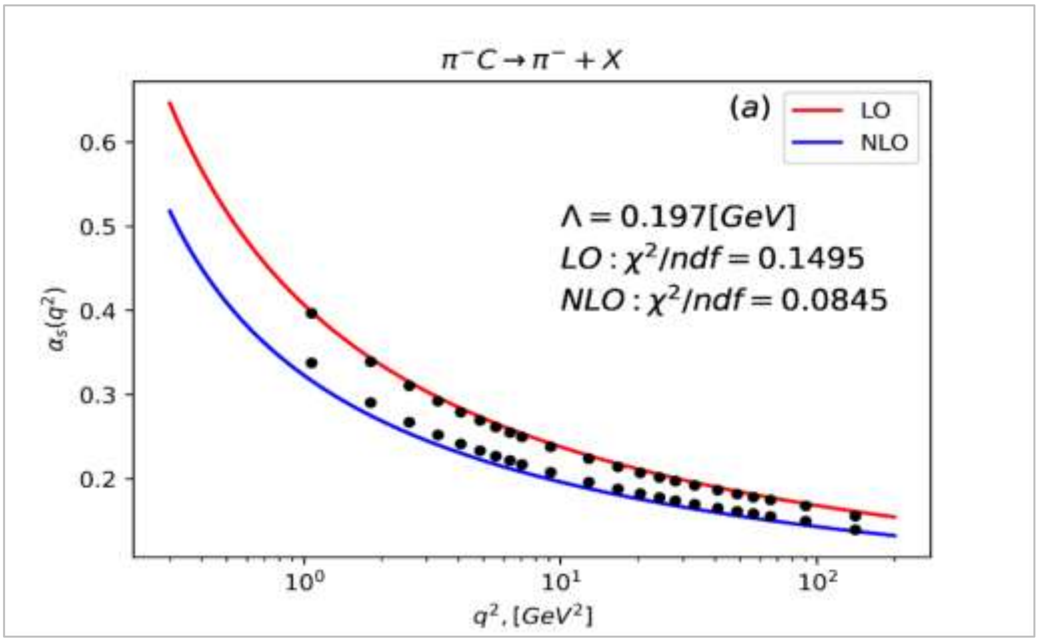

Figure 3a. ***$\alpha_s(q^2)$ dependence of π^- -mesons***

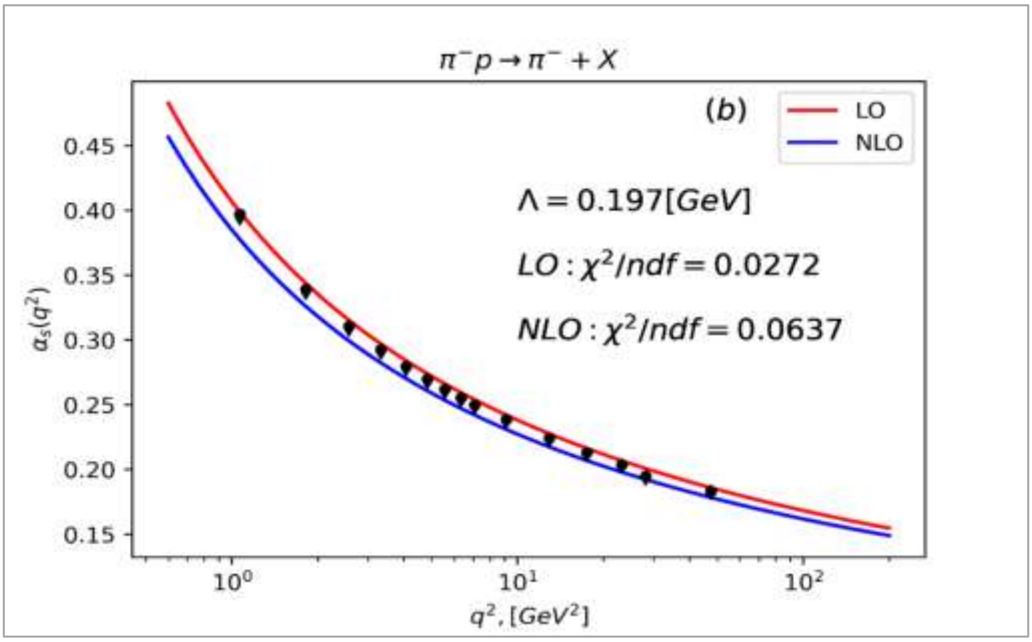

Figure 3b. ***$\alpha_s(q^2)$ dependence of π^- -mesons***

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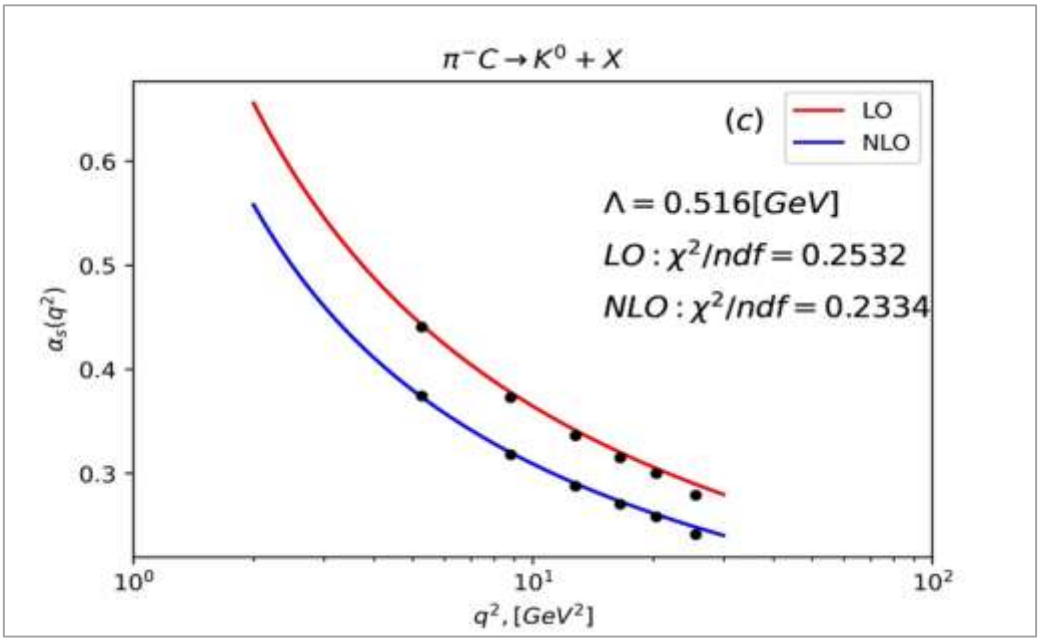

Figure 3c. ***$\alpha_s(q^2)$ dependence of K^0-mesons***

CONCLUSIONS

By using Formula (1), the cut parameter is given value for the particles produced in the reaction. With increasing mass m_a and m_c (according to Formula (1)), the cut parameter Λ_{ac} is increased by the linear dependences (Formula (8) and (9)) with the same slope parameter tgα=$\frac{1}{\sqrt{2}}$ (or $\alpha \approx 35.26^0$). Note that thisis valid for all massive particles.This solves the fact that the cut parameter, which takes a value between 0.1 and 0.5 cannot get an exact value. Dependences of the strong coupling constants, $\alpha_s(q^2)$ are well described by the values, of the cut parameters given in this paper's (Formula (1)). Having such a formula allows researchers to easily use the coupling constant.

The experimental data have also been compared with novel theoretical predictions at leading order and next-to-leading order perturbative QCD. The cut parameter has not previously been calculated using particle mass like Formula 1, and this opens up the possibility to pursue this type of research in high energy physics.

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