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Comparison of Nash and Berge equilibriums in the Bimatrix game

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Abstract: Game theory has numerous applications in applied mathematics, economics, and decision theory. There are several books and articles that deal with Nash and Berge equilibriums. To our knowledge, there are no comparisons or conclusive results related to the optimal decision-making between Nash and Berge equilibriums. We provide numerical experiments for both equilibria.

Key words: Nash equilibrium, Berge equilibrium, global solution, global optimal condition

1. Introduction

Game theory is a critical tool for studying strategic decision-making. It gives a framework for comprehending the activities and effects of rational humans interacting with one another. Equilibrium is a basic notion in game theory that depicts a stable situation in which no party has an incentive to unilaterally break from their existing strategy. There have been several publications devoted to game theory (for example, [5, 6, 16–21, 24, 27, 32, 34, 36–39]).

Nash equilibrium [1, 10, 25, 26, 28] and Berge equilibrium [1, 2, 7–9, 15, 29, 35] are two forms of equilibrium that are widely investigated. Nash equilibrium, named after the mathematician John Nash, arises when each player's strategy is optimal given the other player's strategy. Berge equilibrium, named after the mathematician Claude Berge, is, on the other hand, an extension of Nash equilibrium that allows for the possibility of mixed strategies.

The purpose of this work is to compare and contrast Nash and Berge equilibria in the context of non-zero-sum two-person games, which were also examined in [28, 31, 32], for three [11], five [12], and N-person [13] games using global optimization approaches to establish a Nash equilibrium, commonly known as bimatrix games. The Berge equilibrium was investigated in [14], which offered a concept for a new equilibrium called anti-Berge. There is a scarcity of comparative studies on Nash and Berge equilibriums in terms of player payoffs. Players make individual decisions in a Nash equilibrium, but in a Berge equilibrium, players cooperate to support each other.

We will use numerical experiments to investigate the attributes and consequences of these equilibrium theories. We intend to obtain insight into player behavior and the potential distinctions between Nash and Berge equilibriums by undertaking these tests.

Under certain conditions, the payoff values of the players in the Berge equilibrium exceed those of the corresponding Nash equilibrium outcomes.

The following is the format of the paper: In Section 2, we created a bimatrix game model to study Nash and Berge equilibria.

This model was then modified as a collection of global optimization problems with linear constraints. Section 3 is devoted to conducting numerical experiments on various bimatrix games in order to examine the Nash and Berge equilibrium. Following that, the results are thoroughly compared. Overall, this research improves our knowledge of equilibrium solutions in game theory and provides important conclusions for strategic decision-makers.

2. Methodology

2.1. Nonzero-Sum Two-Person Game (Bimatrix Game)

This section will examine nonzero-sum two-person games, regularly known as bimatrix games. In comparison to zero-sum games, both players can choose to win or lose value at the same time. Bimatrix games show strategic interactions better because they capture an expanded variety of outcomes and strategies.

We investigate bimatrix games using the concept of Nash equilibrium, which is a key solution concept in game theory. When each player's approach is the optimal response to the other player's plan, Nash equilibrium is reached, resulting in a situation in which no party has an incentive to unilaterally depart from their chosen strategy. We can gain significant insight into strategic interactions and potential outcomes by discovering Nash equilibria in bimatrix games.

Consider the bimatrix game with mixed strategies and matrices (A, B) for players 1 and 2.

$$A = (a_{ij}), B = (b_{ij}), i = 1, \dots, m, j = 1, \dots, n.$$

Denote by X and Y the sets

$$X = \{x \in \mathbb{R}^m \mid \sum_{i=1}^{m} x_i = 1,\ x_i \geq 0,\ i = 1, \dots, m\},$$

$$Y = \{y \in \mathbb{R}^n \mid \sum_{j=1}^{n} y_j = 1,\ y_j \geq 0,\ j = 1, \dots, n\}.$$

A mixed strategy for player 1 is a vector $x = (x_1, x_2, \dots, x_m)^T \in X$ representing the probability that player 1 uses a strategy i. Similarly, the mixed strategies for Player 2 is $y = (y_1, y_2, \dots, y_n)^T \in Y$. Their expected payoffs are given by:

$$f_1(x, y) = x^T Ay, (x, y) \in D \triangleq X \times Y,$$
$$f_2(x, y) = x^T By, (x, y) \in D \triangleq X \times Y.$$

We noticed the Nash and Berge equilibrium definitions following.

Definition 2.1. [26] A pair of mixed strategies $x^* \in X,\ y^* \in Y$ is a Nash equilibrium if

$$\begin{cases} f_1(x^*, y^*) \geq f_1(x, y^*),\ \forall x \in X, \\ f_2(x^*, y^*) \geq f_2(x^*, y),\ \forall y \in Y. \end{cases}$$

Definition 2.2. [7] A pair of mixed strategies $\bar{x} \in X,\ \bar{y} \in Y$ is a Berge equilibrium if

$$\begin{cases} f_1(\bar{x}, \bar{y}) \geq f_1(\bar{x}, y),\ \forall y \in Y, \\ f_2(\bar{x}, \bar{y}) \geq f_2(x, \bar{y}),\ \forall x \in X. \end{cases}$$

Existence theorems for Nash and Berge equilibria were studied and proven in [26] and [14, 15]. In [12] and [15] we formulated and proved the following series of theorems:

Theorem 2.1. *[12] A pair strategy $(x^*, y^*) \in X \times Y$ is a Nash equilibrium if and only if*

$$f_1(x^* y^*) \geq [Ay^*]_i\,, \quad i = 1, 2, \ldots, m, \tag{2.1}$$

$$f_2(x^*, y^*) \geq \left[{x^*}^T B\right]_j\,, \quad j = 1, 2, \ldots, n. \tag{2.2}$$

Theorem 2.2. *[15] A pair strategy $(\bar{x}, \bar{y}) \in X \times Y$ is a Berge equilibrium if and only if*

$$f_1(\bar{x}, \bar{y}) \geq \left[\bar{x}^T A\right]_j\,, \quad j = 1, 2, \ldots, n, \tag{2.3}$$

$$f_2(\bar{x}, \bar{y}) \geq [B\bar{y}]_i\,, \quad i = 1, 2, \ldots, m. \tag{2.4}$$

Theorem 2.3. *[12] A pair strategy (x^*, y^*) is a Nash equilibrium for the nonzero sum two-person game if and only if there exist scalars (p^*, q^*) such that (x^*, y^*, p^*, q^*) is a solution to the following optimization problem :*

$$\max_{(x,y,p,q)} F(x, y, p, q) = \langle x^T (A + B) y \rangle - p - q \tag{2.5}$$

subject to:

$$[Ay^*]_i \leq q, \; i = 1, \ldots, m, \tag{2.6}$$

$$\left[{x^*}^T B\right]_j \leq p, \; j = 1, \ldots, n, \tag{2.7}$$

$$\sum_{i=1}^{m} x_i = 1, \; x_i \geq 0, \; i = 1, \ldots, m, \tag{2.8}$$

$$\sum_{j=1}^{n} y_j = 1, \; y_j \geq 0, \; j = 1, \ldots, n. \tag{2.9}$$

Theorem 2.4. *[14] A dual strategy $(\bar{x}, \bar{y})$ is a Berge equilibrium for the nonzero sum two-person game if and only if there exist scalars $(\bar{p}, \bar{q})$ such that $(\bar{x}, \bar{y}, \bar{p}, \bar{q})$ is a solution to the following optimization problem :*

$$\max_{(x,y,p,q)} \Phi(x, y, p, q) = \langle x^T (A + B) y \rangle - p - q \tag{2.10}$$

subject to:

$$\left[\bar{x}^T A\right]_j \leq p, \; j = 1, \ldots, n, \tag{2.11}$$

$$[B\bar{y}]_i \leq q, \; i = 1, \ldots, m, \tag{2.12}$$

$$\sum_{i=1}^{m} x_i = 1, \; x_i \geq 0, \; i = 1, \ldots, m, \tag{2.13}$$

$$\sum_{j=1}^{n} y_j = 1, \; y_j \geq 0, \; j = 1, \ldots, n. \tag{2.14}$$

We created a method to identify Nash and Berge equilibrium in a bimatrix game, as previously discussed and defined.

3. The results

3.1. Computational Experiments

In this section, we provide the findings of numerical experiments conducted to compare the Nash and Berge equilibriua in a game-theoretic foundation. Our goal is to acquire consent into the contrasts between these two equilibrium values.

The problems (2.5)-(2.9) and (2.10)-(2.14) are also nonconvex, so we reduce them into a DC (difference of convex functions) optimization problems, then apply local and global methods and algorithms developed by A.S. Strekalovsky [31].

The results of the numerical experiments presented below were obtained on a personal computer with the following characteristics:

- Intel Core i5-2400MHz, 16Gb DDR4
- used compiler - gcc-5.4.0

Remark 3.1. Note that the condition $F(x^*, y^*, p^*, q^*) = 0$ and $\Phi(\bar{x}, \bar{y}, \bar{p}, \bar{q}) = 0$ is necessary and sufficient for a (x^*, y^*) and $(\bar{x}, \bar{y})$ to be a Nash and Berge equilibrium, p^* is the value of the first player's Nash equilibrium, q^* is the second player's Nash equilibrium; $\bar{p}$ is the value of the first player's Berge equilibrium, $\bar{q}$ is the second player's Berge equilibrium, which will be used in the proposed global search algorithm.

Problem 1. Let $m = 3, n = 6,$

$$A = \begin{pmatrix} 3 & 8 & 14 & 2 & 7 & 19 \\ 5 & 1 & 16 & 4 & 11 & 3 \\ 20 & 6 & 15 & 13 & 10 & 8 \end{pmatrix},$$

$$B = \begin{pmatrix} 14 & 1 & 7 & 20 & 12 & 5 \\ 8 & 7 & 2 & 13 & 11 & 9 \\ 3 & 19 & 6 & 10 & 4 & 11 \end{pmatrix}.$$

Solutions: Nash equilibrium points were found:
$x^* = (0, 0.4545, 0.5455, 0)^T, \quad y^* = (0, 0, 0.2222, 0.7778, 0, 0)^T,$
$p^* = 12.6667, \quad q^* = 4.2727, \quad F = -3.3327 \cdot 10^{-15}.$
Berge equilibrium points were found:
$\bar{x} = (0.5833, 0.4167, 0)^T, \quad \bar{y} = (0, 0, 0.8571, 0, 0, 0.1429)^T,$
$\bar{p} = 14.4167, \quad \bar{q} = 6.7143, \quad \Phi = 0.21131 \cdot 10^{-15}.$

Problem 2. Let $m = 4, n = 8,$

$$A = \begin{pmatrix} 5 & 39 & 4 & 7 & 48 & 42 & 16 & 38 \\ 35 & 16 & 8 & 2 & 19 & 22 & 13 & 30 \\ 2 & 41 & 47 & 6 & 1 & 28 & 13 & 21 \\ 8 & 21 & 47 & 1 & 19 & 36 & 15 & 32 \end{pmatrix},$$

$$B = \begin{pmatrix} 48 & 13 & 23 & 4 & 4 & 26 & 45 & 47 \\ 27 & 12 & 16 & 7 & 24 & 14 & 18 & 27 \\ 24 & 33 & 3 & 30 & 43 & 26 & 50 & 49 \\ 48 & 25 & 42 & 2 & 41 & 38 & 13 & 28 \end{pmatrix}.$$

Solutions: Nash equilibrium points were found.
$x^* = (0, 1, 0, 0)^T, y^* = (0,\ 0,\ 0.025,\ 0.0975,\ 0,\ 0,\ 0,\ 0)^T,$
$p^* = 2.15, q^* = 7.0, F^* = 2.25 \cdot 10^{-6}.$
Berge equilibrium points were found.
$\bar{x} = (0.8182, 0, 0.1818, 0)^T, \bar{y} = (0, 0, 0, 0, 0, 0, 0, 1)^T,$
$\bar{p} = 39.4545, \quad \bar{q} = 49, \quad \Phi = 0.760909 \cdot 10^{-15}.$

Problem 3. Let $m = 5, n = 9$,

$$A = \begin{pmatrix} 11 & 10 & 22 & 19 & 20 & 42 & 8 & 30 & 43 \\ 44 & 39 & 19 & 8 & 27 & 38 & 15 & 9 & 8 \\ 7 & 39 & 44 & 18 & 41 & 30 & 33 & 40 & 31 \\ 11 & 45 & 35 & 33 & 27 & 26 & 6 & 18 & 15 \\ 13 & 47 & 42 & 32 & 13 & 34 & 17 & 41 & 42 \end{pmatrix},$$

$$B = \begin{pmatrix} 41 & 15 & 19 & 26 & 21 & 41 & 21 & 44 & 50 \\ 2 & 12 & 18 & 19 & 42 & 1 & 41 & 1 & 1 \\ 18 & 4 & 36 & 39 & 41 & 39 & 20 & 16 & 37 \\ 44 & 13 & 17 & 31 & 25 & 42 & 23 & 41 & 4 \\ 20 & 38 & 50 & 30 & 28 & 45 & 26 & 30 & 15 \end{pmatrix}.$$

Solutions: Nash equilibrium points were found:
$x^* = (1,0,0,0,0)^T$, $y^* = (0,1,0,0,0,0,0,0,0)^T$,
$p^* = 10.0$, $q^* = 15.0$, $F = 3.5527 \cdot 10^{-15}$.
Berge equilibrium points were found:
$\bar{x} = (1,0,0,0,0)^T$, $\bar{y} = (0,0,0,0,0,0,0,0,1)^T$,
$\bar{p} = 43$, $\bar{q} = 50$, $\Phi = 9.3 \cdot 10^{-15}$.

Problem 4. Let $m = 6, n = 10$,

$$A = \begin{pmatrix} 36 & 23 & 44 & 50 & 7 & 3 & 26 & 48 & 48 & 36 \\ 1 & 35 & 31 & 34 & 13 & 11 & 37 & 42 & 41 & 35 \\ 35 & 3 & 25 & 40 & 33 & 23 & 45 & 19 & 35 & 4 \\ 18 & 29 & 29 & 32 & 48 & 47 & 40 & 17 & 33 & 3 \\ 42 & 42 & 50 & 19 & 40 & 33 & 30 & 18 & 2 & 12 \\ 30 & 50 & 48 & 48 & 17 & 41 & 35 & 43 & 35 & 25 \end{pmatrix},$$

$$B = \begin{pmatrix} 20 & 11 & 46 & 11 & 1 & 40 & 7 & 21 & 16 & 38 \\ 5 & 20 & 46 & 34 & 34 & 17 & 28 & 37 & 5 & 24 \\ 44 & 50 & 44 & 45 & 49 & 26 & 7 & 39 & 10 & 15 \\ 2 & 35 & 29 & 8 & 29 & 2 & 11 & 8 & 12 & 32 \\ 24 & 7 & 27 & 26 & 45 & 33 & 3 & 22 & 1 & 14 \\ 4 & 46 & 8 & 26 & 13 & 14 & 19 & 20 & 11 & 20 \end{pmatrix}.$$

Solutions: Nash equilibrium points were found:
$x^* = (0,1,0,0,0,0)^T$, $y^* = (0.4875,0,0,0,0,0,0,0,0.5125,0)^T$,
$p^* = 21.5$, $q^* = 5.0$, $F^* = 1.0658 \cdot 10^{-14}$.
Berge equilibrium points were found:
$\bar{x} = (0.8378,0,0,0,0,0.1622)^T$, $\bar{y} = (0,0,1,0,0,0,0,0,0,0)^T$,
$\bar{p} = 44.973$, $\bar{q} = 46.0$, $\Phi = 8.43 \cdot 10^{-14}$.

Problem 5. Let $m = 9, n = 15$,

$$A = \begin{pmatrix} 36 & 6 & 22 & 7 & 19 & 46 & 40 & 5 & 17 & 47 & 20 & 47 & 6 & 38 & 7 \\ 5 & 27 & 24 & 44 & 22 & 2 & 6 & 34 & 34 & 36 & 10 & 26 & 11 & 21 & 3 \\ 37 & 24 & 20 & 31 & 40 & 4 & 16 & 25 & 4 & 36 & 37 & 10 & 40 & 36 & 46 \\ 32 & 2 & 38 & 17 & 16 & 41 & 38 & 49 & 6 & 48 & 25 & 11 & 19 & 39 & 42 \\ 35 & 1 & 14 & 44 & 23 & 43 & 12 & 31 & 35 & 7 & 46 & 18 & 41 & 47 & 33 \\ 44 & 49 & 9 & 42 & 33 & 19 & 18 & 21 & 33 & 29 & 5 & 18 & 48 & 33 & 19 \\ 31 & 2 & 43 & 24 & 24 & 39 & 20 & 5 & 9 & 18 & 40 & 37 & 27 & 9 & 49 \\ 42 & 44 & 15 & 22 & 39 & 14 & 23 & 15 & 42 & 26 & 5 & 4 & 38 & 2 & 39 \\ 11 & 21 & 26 & 5 & 1 & 41 & 27 & 29 & 2 & 22 & 40 & 20 & 43 & 19 & 29 \end{pmatrix},$$

$$B = \begin{pmatrix} 47 & 42 & 46 & 45 & 39 & 18 & 3 & 27 & 16 & 6 & 8 & 49 & 18 & 41 & 43 \\ 19 & 23 & 21 & 19 & 47 & 19 & 8 & 8 & 15 & 47 & 14 & 5 & 20 & 28 & 18 \\ 18 & 30 & 33 & 5 & 1 & 25 & 19 & 38 & 20 & 15 & 12 & 33 & 41 & 20 & 36 \\ 28 & 15 & 19 & 49 & 44 & 40 & 33 & 46 & 45 & 13 & 22 & 40 & 30 & 47 & 15 \\ 11 & 38 & 24 & 43 & 46 & 23 & 31 & 23 & 17 & 16 & 7 & 30 & 21 & 16 & 31 \\ 23 & 23 & 22 & 49 & 6 & 21 & 7 & 3 & 41 & 33 & 3 & 11 & 31 & 21 & 9 \\ 46 & 34 & 19 & 26 & 49 & 10 & 23 & 27 & 48 & 1 & 13 & 17 & 12 & 18 & 7 \\ 7 & 49 & 13 & 26 & 50 & 21 & 14 & 4 & 36 & 19 & 10 & 20 & 36 & 49 & 42 \\ 5 & 22 & 15 & 44 & 31 & 3 & 6 & 13 & 28 & 12 & 41 & 32 & 4 & 32 & 39 \end{pmatrix}.$$

Solutions: Nash equilibrium points were found:
$x^* = (0,0,1,0,0,0,0,0,0)^T$, $\quad y^* = (0.5252,0,0,0,0,0,0,0.446,0.0288,0,0,0,0,0,0)^T$,
$p^* = 18.7698$, $\quad q^* = 9.1748$, $\quad F = 3.5759 \cdot 10^{-8}$.
Berge equilibrium points were found.
$\bar{x} = (0,0.2524,0,0,0.1068,0,0,0,0.6408)^T$,
$\bar{y} = (0,0,0,0,0,0,0,0,0,0,0,0.5455,0,0.4545)^T$,
$\bar{q} = 46.0$, $\quad \bar{p} = 38.7273$, $\quad \Phi = 7.8181 \cdot 10^{-8}$.

4. Conclusions

We tested Nash and Berge equilibriums for problems (2.5)-(2.9) and (2.10)-(2.14). We began by providing a definition of the topic, emphasizing the significance of equilibrium notions in game theory.

Problem	(p,q)	Nash	Compare	Berge
3×6	$p =$	12.6667	$<$	14.4167
	$q =$	4.2727	$<$	6.7143
4×8	$p =$	2.15	$<$	39.4545
	$q =$	7	$<$	49
5×9	$p =$	10	$<$	43
	$q =$	15	$<$	50
6×10	$p =$	21.5	$<$	44.973
	$q =$	5	$<$	46
9×15	$p =$	18.7698	$<$	46
	$q =$	9.1748	$<$	38.7273

In all of the above examples, the payoffs of players at Nash equilibrium were found to be lower than those at Berge equilibrium. This finding supports our notion. It is important to point out, however, that this result is an outcome of numerical experimentation rather than a demanding theoretical formulation.

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Биматрицан тоглоом дахь Нэш, Бержийн тэнцвэрийн харьцуулалт

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Хураангуй: Тоглоомын онол нь эдийн засаг, шийдвэр гаргалтын онол, бизнес, улс төр, хэрэглээний математик зэрэг салбарт хэрэглээ ихтэй. Бержийн тэнцвэрийн талаар хэд хэдэн судалгаа, зохиолууд байдаг боловч бидний одоогийн судалснаар түүний Бержийн тэнцвэрийн оновчтой шийдийн хувьд Нэшийн тэнцвэртэй харьцуулсан судалгааны ажил байхгүй байна. Бидний ажил нь энэ харьцуулалтыг хийж Берж ба Нэшийн тэнцвэрүүдийн хувьд тоглогчдын хожлын утгын хувьд харьцуулсан дүгнэлт гаргах зорилготой. Тоон туршилт хийж үр дүнг гаргасан.

Түлхүүр үгс: Бержийн ба Нэшийн тэнцвэр, локал ба глобал оновчтой шийд, глобал оновчтой нөхцөл