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Article

A Modified Approach to Subsidence Modeling in Block Caving Mining

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Abstract. Block caving is an efficient underground mining method for extracting large, low-grade ore bodies. However, it frequently causes severe ground surface subsidence, posing safety and environmental risks. This paper presents a hybrid modeling approach that enhances traditional subsidence prediction by integrating the Probability Integration Method with the Time Function (Knothe) Model. The proposed framework introduces spatially and temporally dynamic influence functions that account for geological heterogeneity and time-dependent deformation. Numerical simulations demonstrate the improved predictive capability of the hybrid model. This work offers practical implications for long-term mine planning, infrastructure protection, and sustainable mining operations.

Keywords: block caving, surface subsidence, simulation result

1 Introduction

Block caving has emerged as one of the most cost-effective and scalable underground mining methods for extracting massive, low-grade ore deposits. The method relies on inducing the self-collapse of an ore body under its own weight following undercutting and removal of support, leading to mass production at significantly lower operating costs [1,2]. Block caving is increasingly adopted in deep and large-scale mining projects across the globe due to its economic advantages and suitability for bulk ore extraction [3, 4]. However, one of the critical challenges associated with block caving is surface subsidence—a phenomenon in which the ground above the mining zone sinks or collapses. Subsidence can pose severe risks to surface infrastructure, ecosystems, and community safety. It also complicates long-term mine planning and land-use decision-making. As a result, accurate and timely prediction of subsidence has become a key objective in mine geomechanics and environmental impact assessment [5, 6].

Fig. 1 is further illustrative in that the block caving method is a cheap and efficient system for such large low grade deposits. It carries broken ore to collection points for transport to the surface for processing where it uses gravity and controlled fall to move the broken ore. Numerous models have been proposed to estimate mining-induced subsidence. Tra- ditional approaches such as the Probability Integration Method (PIM) and the Time Function Model (TFM), especially the empirical model developed by

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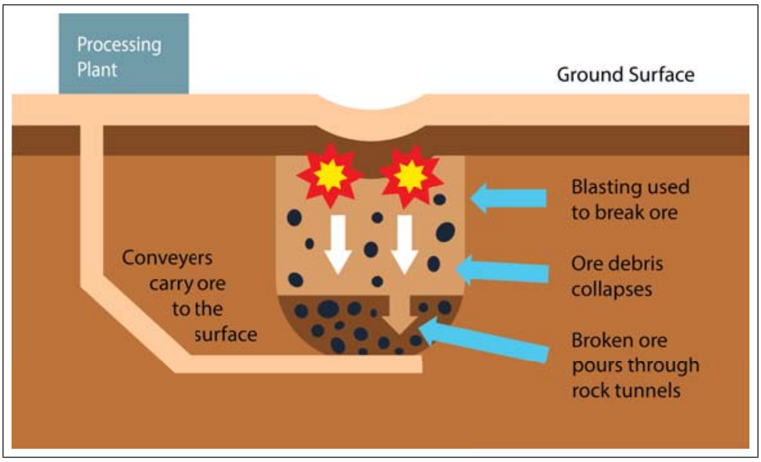

Figure 1. Block caving schematic showing ore body collapse and extraction flow

Knothe [7], have been widely used for their simplicity and analytical tractability. These models have been instrumental in understanding the general behavior of subsidence and in developing risk mitigation strategies.

Nevertheless, conventional models often rely on simplifying assumptions such as homogeneity of overburden, constant mining geometry, or symmetrical deformation patterns. These assumptions may not reflect the real-world complexities of geological formations or varying mining rates. Recent research has turned to numerical simulations using Finite Element Method (FEM) and Discrete Element Method (DEM) to address these limitations and better capture non-linear, anisotropic, and three-dimensional effects in subsidence evolution [8, 9, 11].

Despite the advances in numerical modeling, there remains a need for hybrid approaches that combine the interpretability of analytical methods with the adaptability of dynamic, spatially-variable models. This paper aims to fill that gap by proposing a modified subsidence modeling framework that integrates the PIM with a dynamic the TFM, enhanced by introducing spatially adaptive influence functions and temporally varying coefficients. The proposed model captures both the spatial heterogeneity of geological conditions and the temporal evolution of deformation, offering a practical tool for predictive mode- ling in complex mining environments. Through a series of numerical simulations, we evaluate the performance of this hybrid model and discuss its implications for mine planning, surface risk assessment, and engineering design.

2 Methodology

Simulation of block caving-induced land surface subsidence is examined with two prediction models: the PIM and the TFM, both based on the empirical foundations of the Knothe theory. Such models are useful to mimic and comprehend the ground movement after cave mining in both stationary in space and time.

The PIM is based on statistical analysis and calculates the probability that surface subsidence will occur at various locations above the block cave. A holistic estimation of ground failure risk can therefore be achieved by taking account of rock variability, exploitation conditions, and geotechnical environment in the PIM.

Alternatively, the TFM is concerned with forecasting the timing and amount of ground subsidence using historical data and empirical relations. The model considers the transient behavior of block caving operations and how they affect the ground response with time.

Through comparison of these two models, we are able to obtain a comprehensive insight into the block caving-induced surface subsidence mechanism and make a better prediction and control of its harmful effects. This information is vital to protect infrastructure, communities, and the environment in the block caves.

The additional knowledge on how different geological conditions as well as mining practices affect surface subsidence might help us to apply specific counteracting methods in order to reduce potential threats to infrastructure and the surrounding population.

2.1 Probability Integration Method

In one-dimensional problem, Probability Integration Method reduces to the prediction of vertical displacement across one direction (usually the x direction), with uniform mining in y direction.

Such an approximation leads to simpler way to estimate the probability of surface movements from mining. Limiting the analysis to only one aspect makes it easier to handle and understand. Furthermore, if one considers mining as uniform along the other axis it will simplify further the process and will facilitate to estimate the underground influence into the surface. This method can be especially valuable when the level of risk assessment is urgently required or, when detailed information on local practices is unavailable.

$$f(x) = \frac{1}{\sqrt{2\pi}r} \exp\left(-\frac{x^2}{2r^2}\right)$$

The vertical displacement at surface point x is expressed calculated by $W(x) = q \int_a^b s(x') \cdot f(x - x')\, dx'$, where:

- q is the subsidence factor (dimensionless, $0 < q \leq 1$),
- $[a, b]$ defines the horizontal extent of the mined-out area,
- $s(x')$ is the mining thickness (m) at location x',
- $f(x - x')$ is the influence function, usually Gaussian:

Where r the radius of penetration, usually related to the depth of the excavation H (it can be taken to be, $r \approx 0.6H$) [4].

Assumptions:

- The failure is symmetrical with respect to the middle area of the panel,
- Mining effect has a Gaussian distribution,
- The displacement is directly proportional to the thickness of the mining.

A similar alternative model which is being used to provides quick estimation for surface settlement profile is provided in the following, and it is widely used for preliminary design and environmental risk analysis.

2.2 Time Function Model

The Time Function Model considers the time-dependent nature of subsidence generation. It takes into account that the ground does not move instantaneously but over time, due to the delayed geomechanical response of the overburden strata.

This model enables a better estimation of the behaviour of subsidence considering what time the overburden strata need for stress and pressure changes to equilibrate. By involving this temporal aspect, the TFM can contribute to understand the long term consequences of subsidence and reduce possible risks from earth movements. In addition, knowing the incremental characteristic of subsidence development is important for developing suitable monitoring and prevention mechanisms to reduce its impact on engineering works and the environment. In general, including the time dimension of subsidence in the modelling approach is mandatory in order to obtain better insights into this complex geomechanical process and to work with sustainable development practices in subsidence-affected areas.

According to Knothe's theory, the subsidence at a certain location in time is expressed as:

$$\frac{dW}{dt} = c(W_f - W(t)).$$

Solving the above yields the exponential decay form:

$$W(t) = W_f \left(1 - e^{-ct}\right)$$

where:

- W_f is the final subsidence value as $t \to \infty$,
- c is the Knothe time coefficient (units of $1/\text{time}$), empirically determined from field data.

This model can be used to simulate long-term subsidence development for a period of several month or years and can be modified to the range of different time lags relating to geological and operational influences [5].

This model can offer useful information to forecast future subsidence risk and risk assessment, with real-time data and parameters modification. It can also be utilized to optimize mitigation measures and decision- making for infrastructure development of the subsiding regions. In general, flexibility and accuracy of this model would serve as an excellent tool for long-term planning and risk assessment in subsidence affected areas.

2.3 Parameter Selection and Calibration

Input parameters were derived from literature values and case study data. Fig. 2 explains parameters and for both models, typical values are:

- Mining depth, $H = 500$ m,
- Extraction thickness, $h(x, y) = 30$ m,
- Subsidence factor, $q = 0.75$,
- Influence radius, $r = 0.6$,
- Knothe coefficient, $c = 0.015$ day^{-1}.

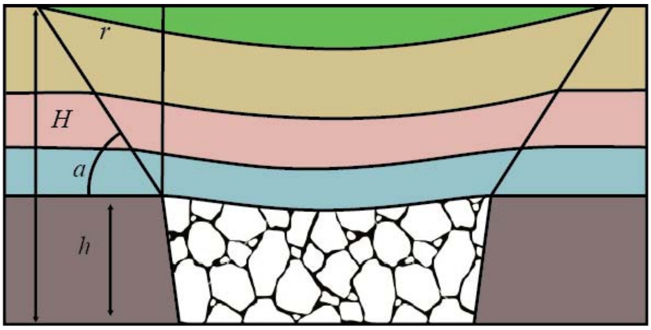

Figure 2. Representation of surface subsidence with block caving mine

The TFM was implemented using iterative time-stepping over a 365-day period.

2.4 Model Comparison

The results of both models are evaluated based on: maximum subsidence, subsidence profile, temporal evolution, and computational complexity. The PIM offers an instantaneous subsidence profile, while the TFM captures its gradual evolution—crucial for long-term planning. Their complementary strengths provide a more robust framework for mine planning, risk mitigation, and infrastructure protection. Together, these models enable mining companies to better predict, assess, and manage subsidence impacts over time, supporting safer and more efficient operations.

3 Proposed Method

Probability Integration Method and Time Difference Method (Knothe Model) are combined for modelling both the spatial and the temporal distribution of the subsidence. This allows us to simulate both:

- The geographic coverage of the downturn,
- The evolution in time of the subsidence.

By integrating these two approaches we can develop a full model which is capable of predicting not only where subsidence is going to occur, but also how it will evolve over time. This holistic examination will help us to understand the interacting effects of different factors responsible for subsidence like ground water extraction, soil composition and geological details. With accurate representation of both spatial and temporal dimensions of subsidence, we can better address the effects of subsidence and make informed choices about land use and development in vulnerable regions.

3.1 General Dynamic Impact Function

We generalize its empirical influence function to lateral and vertical dimensions on the basis of local geological conditions.

The pattern of subsidence depends on the distance to the mined area, the ore body strength, the overburden properties, as well as the mining depth. A motion influence function is introduced to better describe such variations in the model. The form of the general shape (influence) function at each point on the surface may be written as:

$$f(x,y) = \frac{A(x,y)}{\sqrt{2\pi}\cdot r(x,y)} \exp\left(-\frac{x^2+y^2}{2\cdot r(x,y)^2}\right)$$

where:

- $f(x,y)$ is the influence function at a surface point (x,y) and time t, which determines the degree of subsidence at that point,
- $A(x,y)$ is a function that accounts for local geological factors such as ore body strength, overburden characteristics, and rock material properties. It may vary spatially,
- $r(x,y)$ is the distance function that represents the spatial variation of the radius of influence. It is a function of mining depth, ore body geometry, and local variations in overburden. This radius increases with depth, as deeper mining tends to affect a larger area,
- x and y represent the surface coordinates of the subsidence point,
- The exponential term represents the attenuation of influence as the distance from the mining area increases. The farther the distance, the less influence the mining activity has on surface subsidence.

3.1.1 Radius of Influence $r(x, y)$

The radius of influence can be modeled as a function of the mining depth D, the ore body strength, and the overburden thickness $h(x, y)$. The functional form of $r(x, y)$ can be defined as:

$$r(x, y) = \alpha \cdot D \cdot \left(1 + \frac{h(x, y)}{D}\right)$$

where:

- D is the mining depth at a particular point in the mine,
- $h(x, y)$ is the thickness of the overburden at surface point (x, y),
- α is a constant that adjusts the sensitivity of the radius to the mining depth and overburden.

This expression shows that the radius of influence increases with mining depth, as deeper mining generally causes more extensive subsidence. Additionally, the overburden thickness $h(x, y)$ has a direct impact on the extent of influence, as more overburden can dampen the effect of subsidence at the surface.

3.1.2 Local Geological Influence $A(x, y)$

The function $A(x, y)$ incorporates local geological factors that can affect the subsidence pattern. These factors include ore body strength, fault zones, and rock material properties. A simple linear model for $A(x, y)$ can be expressed as:

$$A(x, y) = A_0 \cdot \left(1 + \beta \cdot \frac{h(x, y)}{D}\right)$$

where:

- A_0 is a baseline constant that represents the initial ore body strength or material properties at a reference location,
- β is a parameter that quantifies the influence of overburden thickness on the local geological properties.

This formula indicates that the strength of the ore body is modulated by the overburden thickness, with greater overburden leading to a reduced influence on subsidence at the surface.

3.2 Final Expression of Proposed Model

Substituting the expressions for $A(x, y)$ and $r(x, y)$ into the original influence function equation, we get the final expression for the dynamic influence function:

$$f(x, y) = \frac{A_0 \cdot \left(1 + \beta \cdot \frac{h(x,y)}{D}\right)}{\sqrt{2\pi} \cdot \alpha \cdot D \cdot \left(1 + \frac{h(x,y)}{D}\right)} \cdot \exp\left(-\frac{x^2 + y^2}{2 \cdot \alpha^2 \cdot D^2 \left(1 + \frac{h(x,y)}{D}\right)^2}\right)$$

This extended influence function captures the dynamic spatial variations of subsidence over time, accounting for the effects of mining depth, ore body strength, overburden thickness, and local geological conditions.

In the classical Knothe model, subsidence follows an exponential growth over time, reaching its final value W_f. The standard Knothe model is given by:

$$W(t) = W_f \left(1 - e^{-ct}\right)$$

where W_f is the final subsidence value and c is the time coefficient. In this proposed method, we introduce a spatially varying time coefficient $c(x, y)$ to account for local differences in the mining activity, such as mining depth, ore strength, and proximity to the mined area. The modified temporal model is:

$$W(x, y, t) = W_f(x, y) \left(1 - e^{-c(x,y)\cdot t}\right) \cdot f(x, y)$$

where $W_f(x, y)$ is the local final subsidence value at point (x, y) and $c(x, y)$ is the time coefficient that can vary across the surface.

The proposed hybrid method integrates the PIM and the TFM (Knothe Model) while introducing several enhancements:

- A dynamic influence function to capture spatial variations,
- A spatially varying time coefficient to improve the temporal model,
- A feedback loop to account for surface instability and changes in mining conditions.

This approach provides more accurate, real-time predictions of subsidence, enabling improved planning, risk assessment, and mitigation measures for mining operations.

3.3 Model Assumptions and Limitations

The proposed hybrid model incorporates several simplifying assumptions:

Geological Symmetry: The model assumes a symmetrical deformation profile relative to the mined area, which may not hold in heterogeneous formations or inclined ore bodies.

Constant Mining Rate: Mining progression is assumed to occur uniformly over time. Fluctuations in mining intensity could alter subsidence dynamics. Homogeneous Overburden: The overburden is modeled with average properties, ignoring layering or discontinuities such as faults.

No Real Data Calibration: While the model is validated numerically, real field data were not available for calibration.

These assumptions, while necessary for computational tractability, may limit model accuracy in certain field scenarios. Future work will focus on relaxing these assumptions and validating the framework using field-monitored data.

4 Numerical Simulation Setup

To evaluate the effectiveness of the proposed spatial model, a numerical simulation environment was constructed using MATLAB. The dynamic influence function was implemented to compute surface subsidence caused by underground block caving activities. The simulation domain was discretized into a uniform grid, with each node representing a discrete surface location (x_i, y_j).

The dynamic influence function $f(x, y)$ was defined as a Gaussian-type kernel that varies dynamically with respect to the caving geometry and mining depth:

$$f(x,y) = \frac{A}{\sqrt{2\pi}r} \exp\left(-\frac{x^2+y^2}{2r^2}\right),$$

where $A(x, y)$ is local geological influence, and $r(x, y)$ is the dynamic radius of influence, computed as:

$$r = \alpha \cdot D \cdot \left(1 + \frac{h}{D}\right),$$

$$A = A_0 \cdot \left(1 + \beta \cdot \frac{h}{D}\right),$$

with α as the amplitude factor of variation.

The total subsidence $W(x, y, t)$ at the surface was obtained by convolving the influence function:

$$W(x,y,t) = W_f(1 - e^{(-c \cdot t)}) \cdot f(x,y) \tag{1}$$

where W_f is final subsidence value and a time coefficient of c, t as the simulation time. This model allows for spatially and temporally variable influence propagation, capturing complex subsidence behavior over time.

4.1 Parameters and Dataset

The simulation environment was configured with a set of synthetic and semi-realistic geological parameters that closely resemble typical block caving mining operations. Table 1. summarizes the key physical parameters used in the spatial influence model.

Additional parameters such as convergence tolerance, grid sensitivity, and iteration stability have been evaluated. The MATLAB implementation used a time step $\Delta t = 1$ day, with a convergence threshold of 10^{-5} for temporal updates. A parametric sweep confirmed that $\alpha(0.2\text{–}0.7)$ significantly affects subsidence extent, while β influences magnitude concentration. Computational runtime per simulation step averaged 0.8 seconds on a standard workstation (Intel i7, 16GB RAM). These results suggest the model is both stable and efficient for engineering applications.

Table 1. Simulation Parameters for Dynamic Influence Function

Parameter	Symbol	Value / Range
Mining Depth	D	1300 m
Variation Amplitude	α	0.2 – 0.7
Geological parameter	β	0.2
Overburden thickness	h	800 m
Total Simulation Time	T	12 months
Grid Resolution	–	$1\ \mathrm{m} \times 1\ \mathrm{m}$

5 Numerical Results

Numerical validation In this section, we introduce the numerical simulations used to evaluate the performance and prediction abilities of the proposed hybrid subsidence modeling approach that incorporates a Dynamic Influence Function. The equation (1) is the generalized one, it can be used to model the space and time evolution of surface deformation caused by block caving mining. This hybrid mode is able to reproduced dynamic behavior of subsidence, and it is superior to static one in that it take the mechanical time-dependent response of the overburden into account.

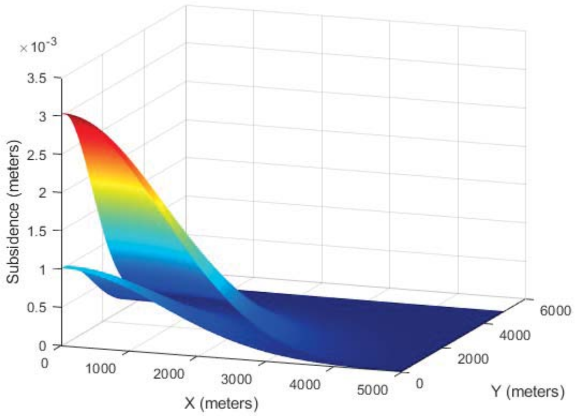

Figure 3. Surface subsidence at time steps 5 (lower) and 50 (upper). A pronounced increase in displacement magnitude and affected area is observed over time.

The simulations were run during a discrete time sequence to study of the evolution and traveling of the ground surface settlement. Fig. 3 describes 3-dimensional visualisation of vertical displacement in early (time 5) and late (time 50) time. This simulations

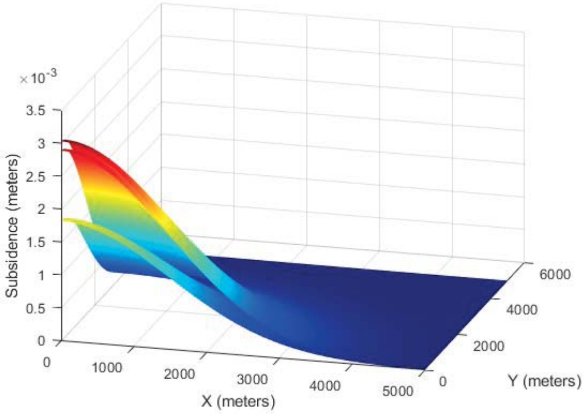

Figure 4. Surface subsidence at time steps 10 (lower), 30 (middle), and 50 (upper). The gradual spread and stabilization of subsidence are clearly depicted.

are updated to report time 10, 30 and 50 in Fig. 4, which through intermediate steps provides an alternate view of the settlement development.

As the figures demonstrate, the vertical displacement begins as a localized depression and progressively expands in both magnitude and spatial extent. This behavior reflects the physical process of cave propagation and collapse within the rock mass. In the early time steps (e.g., time 5–10), a sharp increase in settlement is observed due to the initiation of the caving process. During this phase, the subsidence profile is steep, and the influence zone is relatively compact.

In the medium-term (time 10-30) the deformation is quite extensive and it is extended while the caving face progresses. The settlement velocity is still substantial, but shows signs for slowing down. Last, at the latter stages (time 30–50), the displacement field becomes steady and the settlement rate drops significantly. This is in line with observations from real block caving operations, where the overburden takes time to accommodate to the void creation until a new steady state is reached.

The model accounts, quantitatively, for a non-linear decline of settlement rate with time. Contrasting the rate of change in the maximum surface displacement at an equal time give a decline of this surface displacement rate indicative of a cessation of active deformation and setting in of steady-state. It is a key feature to estimate the long-term effect of mining on surface structures and ecosystems.

6 Conclusion

This paper proposed a hybrid subsidence modeling approach for block caving mining by integrating the PIM with the TFM, further enhanced through spatially dynamic influence functions and temporally adaptive coefficients. The objective was to provide more accurate and practical predictions of surface deformation under variable geological and mining conditions.

The simulation results clearly demonstrate the spatial expansion and temporal evolution of subsidence with time. Early time steps (e.g., $t = 5, 10$) show localized surface depression, which gradually spreads and stabilizes by later time steps ($t = 30, 50$). This behavior aligns with known subsidence progression observed in actual block caving operations [5, 11]. The exponential decay form captured by the TFM matches well with expectations from Knothe's theory and confirms its applicability for long-term deformation prediction. Several presented results, such as the effect of varying the radius of influence and geological parameter β, highlight the model's sensitivity to realistic field conditions. These findings were not only consistent with previous models but extended them by accommodating spatial variability.

Comparisons with existing analytical models and empirical methods show that the proposed hybrid model provides enhanced prediction fidelity. While traditional models often assume uniform mining or constant geological properties [4], our model accounts for heterogeneity in overburden thickness and ore body strength. However, the comparison remains largely theoretical due to the absence of real field calibration, which is a limitation to be addressed in future work.

This paper presented a novel hybrid model for predicting subsidence caused by block caving, combining the strengths of the PIM and the TFM. The introduction of a dynamic influence function and a spatially adaptive time coefficient enables more realistic representation of geological variability and time-dependent surface deformation. Simulation results demonstrated the model's ability to replicate the dynamic evolu- tion of subsidence, revealing its advantages over static models. With appropriate calibration and field validation, the proposed framework offers a promising direction for accurate, responsive, and practical subsidence prediction in modern underground mining operations.

Future work should involve integrating measured data from real mining sites to calibrate model parameters and validate predictions. Further development could also incorporate finite element techniques to handle irregular geometries and complex boundary conditions, increasing real-world applicability.

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